CATEGORIES, PRESHEAVES, SHEAVES AND COHOMOLOGIES FOR THE THEORY OF CONSCIOUSNESS

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An analysis with commentaries on a theory of consciousness, developed recently by Goro Kato is presented. The main pillars of Kato's theory are a reference category and a target category with presheaves that are functors with special properties, between them. The reference category is the generalized time category; the target category contains thoughts and even physical structures. The presheaves are targeting local thoughts, and the coresponding sheaves form global thoughts from local thoughts. The cohomology defined on a sequence of objects in the target category, every object representing a conscious entity (or a person) shows the position and the links of a person (consciousness) in a network of persons (consciousness).

This paper, not being addressed to professional mathematicians, but to those working in consciousness science, psychology, science of information, physics, etc., offers the necessary notions of presheaves, sheaves and cohomology as a general introduction to them, in order to follow the ideas and concepts of the theory of consciousness based thereupon.

At the same time some considerations on the theoretical construction of Kato are presented. For instance, the reference category of the generalized time might indeed be used at the level of a universe, looking from the universe inside it or toward the deep existence of reality. If the reference category were in the deep existence, that has no time, looking from the deepest strata of reality toward the universes and consciousness, y compris the fundamental consciousness of existence, the generalized time category would have to be replaced by another one (perhaps with a form of *cronos*, without duration).

It is shown that perhaps, at least in some cases, it would be possible to work only in the frame of categories with functors among them (of the type of Kato's target category) and using also the cohomology theory. It will remain to be seen if the presheaves and sheaves would be useful in this case. Kato's frame and the above mentioned frame are two possibilities, but in both frames the cohomology might have the same role.

Kato's theory is thought provoking and for both frames mentioned above, an important aspect will be a connection with the phenomenological and structural-phenomenological categories and functors described by, until now, another line of thought that uses categories and functors for consciousness and also for physical processes. It seems that such a connection is indeed possible and will be tried in further works.

INTRODUCTION

Kato and Struppa proposed to use the theory of presheaves and sheaves in the frame of category theory for dealing with the consciousness theory [1], [2]. Struppa, Kafatos, Roy, Kato, Amoroso delivered a general paper on the use of category theory in science and in the consciousness problem [3]. Soon afterward, Goro Kato began to elaborate a theory of consciousness based on the theory of sheaves [4], [5] and also to propose a sheaf theoretic foundations of ontology [6].

Drăgănescu developed a line of thought [7] based on his structural-phenomenological philosophy of science and on the concepts of an integrative science elaborated with Menas Kafatos [8]. These works [9], [10], [11], [12], [13], [14] and others elaborated with Menas Kafatos and Sisir Roy are trying to extend the theory of categories and functors from the structural domain of mathematics and science to the phenomenological and structural-phenomenological domains.

In this paper the theory of Struppa and Kato is examined as developed in the latest works of Kato [4], [5], [6] in the light of the integrative point of view of Kafatos and Drăgănescu.

In [1] it is mentioned that 'The proposal presented here should not be seen in opposition to [15], where the general principles of the structural-phenomenological and integrative science were presented, our note M.D., but rather as a complement and it is our hope that our formalism may in the future be used to support the ideas put forward in [15]'.

THE PILLARS OF KATO'S THEORY

In Kato's theory [1], [2], [4], [5], [6] there are three pillars:

- a 'departure (reference)' category T with an associated topological space *T* (which is or may be seen as a category);
- presheaves (which form a category) between *T* and a category K of values for the presheaves;
- a target category (category of values), K*,* which is built as a product category

$$K = (\Pi C_\alpha)_{\alpha \in \Gamma} \quad (1)$$

where Π is an index set. The index set is possibly uncountable.

A presheaf in the theory of categories [16] is a contravariant functor on a topological space, the topology being seen as a category. In [13] some commentaries were presented on the definition of the topology as

Topological space (shortly Topology) = < set (category), defined structure on the set (category) > (2)

For a reference category T, the topological space *T* is of the form

T = Topological space (shortly Topology) = < T, Grothendieck topology) (3)

where the Grothendieck topology is defined as in [13] (after [17]) or with a more workable definition presented later in this paper, with the purpose to analyze Kato's works. The objects of a topology as a structure of subsets of T, y compris T, may be simple sets, or sets like abelian groups, rings, etc. These form evidently a category of such objects.

A presheaf is a contravariant functor between a topology, seen as a category, and another category, the target category K. The category K may have or not the form (1). The theoretical mathematical condition of the target category is to be a category with infinite direct products [17], that is, if any family of objects (may be infinite) of the target category has at least a direct product. This is the case of categories of sets, of abelian groups (an abelian group is also a set but with some defined structures), etc.

The presheaf being a functor, to an object of T corresponds an object of K,

$$\bar{\mathrm{U}} : T \to \mathrm{K} \tag{4}$$

where Ū means a contravariant functor (for which all the arrows in K are inversed, in opposition to the covariant functor

$$\mathrm{U} : T \to \mathrm{K} \tag{5}$$

That lets the arrows in K to correspond to those in *T).*The contravariant functor may be written [17] as a covariant functor

$$T^{\mathrm{op}} \to \mathrm{K} \tag{6}$$

where T^{op} is the category T with inversed arrows, and in K the arrows are no more inversed.

An object V (which is an open set) of T has a target or a value Ū(v) in K (or exactly U(V), when the arrows are of not primary concern), where U(V) is a mathematical structure reflecting the same structure as V.

For a presheaf, as for any functor, to an object in T corresponds an object in K.

For a morphism (arrow, map) in T which is an inclusion

$$\mathrm{W} < \mathrm{V} \tag{7}$$

The object W being a subset of V, i.e for the morphism in T

$$\alpha : \mathrm{W} \to \mathrm{V} \tag{8}$$

corresponds in K the homomorphism (using the notation P for the presheaf),

$$\mathrm{P(V)} \to \mathrm{P(W)} \tag{9}$$

observing the inversed arrows.

The homomorphisms in K respect the identity morphism

$$1_{\mathrm{v}} : \mathrm{p(v)} \to \mathrm{P(v)} \tag{10}$$

and the composition of morphisms

$$\mathrm{P(Y)} \to \mathrm{P(V)} \to \mathrm{P(W)} = \mathrm{P(Y)} \to \mathrm{P(W)} \tag{11}$$

when Y < V < W. In T,

$$(\mathrm{W} \to \mathrm{V}).(\mathrm{V} \to \mathrm{Y}) = \mathrm{W} \to \mathrm{Y} \tag{12}$$

It may be observed also in (11) the inversed arrows in comparison with (12).

THE GENERALIZED TIME CATEGORY *T*

The reference category *T* is considered by Kato to be the generalized time space or generalized time category [3], [5], [6].The authors of [3] write:

"The time which we, as conscious beings, experience, is a linear, uni-dimensional space, *i.e.* the real line R. This allows us to experience notions such as time arrow, past, present, and future, as well as birth and death. From our point of view *T* is a general topological space (*a priori* not necessarily Euclidean, nor even locally Euclidean). We only ask that there is an embedding

$$i : R \to T \tag{13}$$

so that conscious entities 'live' and are diffused over this generalized time *T*. […] We also note that this model allows for multiple times, through different embeddings of R into *T*. This model is therefore consistent with the many-worlds interpretation of quantum mechanics […] our model does not require, neither does expect, a collapse of the wave function to generate different worlds. In our model, the several worlds exist simultaneously."

A generalized time interval in *T* is an object V in *T*.

The topology of this category is taken to be a Grothendieck topology [2], *i.e.* is a

$$\text{Topological space} = T = (\text{Category T, CovT}) \tag{14}$$

where CovT is a set of families of coverings of objects (the notion of covering will be detailed later in this paper) in CatT or, the same thing, T because T may be seen as a category, CatT. Also *T* may be seen as a category.

In order to have presheaves on such a topology, some minimal requirements are necessary [2], for instance, Cat*T* to be a base set and that there is an embedding of a real line into *T*.

We observe that the reference category of the generalized time might indeed be used at the level of a universe, looking in the universe or even from the universe toward the deep existence of reality.

If the reference category were in the deep existence, it could not be a time because the deep existence has no time. What could such a reference category be? Looking from the deepest strata of reality toward the universes and consciousnesses, y compris the fundamental consciousness of existence, the generalized time category has to be replaced by another category (perhaps with a form of *cronos*, without duration).

The point of departure, concerning the reference category as a generalized time category, might be useful for limited objectives, and indeed these seem to be the important ones. Still this is not a point of departure for the generation of time itself in a universe, together with the physical space of that universe and all most elementary particles.

T does not cover the superuniverse of all the universes in existence and the problem of the reference category has to be extended in an appropriate way without the notion of time. Perhaps the fundamental phenomenological category of the entire existence might be a source for finding a solution in such a case.

THE TARGET CATEGORY K

The target category K is taken by Kato to be a product of categories (1), maybe an *a priori* infinite product of categories [3].

Every category C_α has objects, and an object may have elements, An element is considered to be a thought.

As such, the target category is a world of thoughts distributed in the objects of the categories $(C_\alpha)_{\alpha\in\Gamma}$. These thoughts are in fact considered as given, like Plato ideas. They are selected by a presheaf from *T* to K by the action of a consciousness.

A presheaf is an assignment of an object in K to each open set of *T*. The elements of these sets are called sections [16]. Therefore, the thoughts of a consciousness are sections of its presheaves [2].

The family of categories $(C_\alpha)_{\alpha\in\Gamma}$,pre-existent as we observed before, provides values for the presheaves, these values being thoughts. Kato and Struppa [2]write:

"In our view, a reasonable model for conscious entities is to consider presheaves on such a category (*T*, our note M.D.); the issue becomes of the value set for such presheaves. If we imagine a presheaf to represent, for example, a brain, we are aware of the fact that at any given time, a brain can be consciously focusing on different aspects of reality, and that these aspects may require a totally different structure. We therefore postulate the existence of a family (possible uncountable) of categories C_α (α to run in index set Γ) which can provide values for the presheaves on *T*. Namely, for a presheaf F and for an object V in the category *T*,

$$F(V) = \{\ldots, F_\alpha(V),\ldots\} \tag{15}$$

where $F_\alpha(V)$ is an object in C_α. We will say that F_α is the α-th of F or its α-th projection. Clearly, F_α is itself a presheaf with values in C_α, while F is a presheaf with values in the product category of all the C_α's. On the other hand, when we fix a 'generalized time interval' V in *T* (*i.e.* an object in *T*), the set $F_\alpha(V)$ will denote the 'brain activity' of F, at time V, in the category C_α.

Therefore, we will consider the category $\check{T}$ of all presheaves from *T* to a certain product of categories C_α. Such a model will be our model for the conscious universe (or sea of consciousness)

$$\check{T} = \{F : T \rightarrow \Pi C_\alpha\} \tag{16}$$

The sections of any α-th of a presheaf F are interpreted as the thoughts of the conscious entity represented by F."

It may be seen that a conscious entity *is also a brain* that has thoughts. The presheaves represent rather the *activity of the brain (or of the conscious entity).* The content of the consciousness is in K, it is true, by the sections established by the the presheaves of the conscious entity.

A consciousness is not only the presheaf, but the presheaf with its sections in K. The presheaf, as a functor, represents the internal dynamics of the consciousness and, consequently, is a component of the consciousness. Because this dynamic component points to the content of the consciousness, in an interval of time *T* (an object of *T*) it might be said, as Kato and Struppa did, that the presheave is, freely speaking, the consciousness.

Concerning the target category K, Kato divides [5][6] the categories $(C_\alpha)_{\alpha \in \Gamma}$ into some parts. *The first part* contains the physical world categories, namely C_1 being the microworld and C_2 the macroworld. He adds even the generalized time category *T* noted with C_0. The first part of Γ corresponds to $(C_j)_{j=0,1,2,3,\ldots\ \alpha \in \Gamma}$ because, indeed, other levels of the physical world might be taken into consideration.

All these physical categories are perhaps not only to be used in K, but in themselves because *among such categories there are functors.* For instance, between the physical microworld category and the physical macroworld category there are functors that relate phenomena in the two categories.

As an example, for the physical part of the brain, Drăgănescu and Kafatos [18], [19] considered a chain of physical categories with functors (in both directions) among them. In [18] one considers

$$C_{str} \leftrightarrow C_{1str} \leftrightarrow \ldots \leftrightarrow C_{kstr} \leftrightarrow C_{coher.\ quantum\ waves} \leftrightarrow C_{phen}$$

In the above, C_{1str}, C_{2str}, ... ,C_{kstr} are structural categories of the brain, other than neuronic structures (C_{str}), but intermediary categories (dendritic networks, molecular vibrational fields along protein filaments, perimembraneous waves, quantum cortical fields – after Jibu and Yassue, 1995) between C_{str} and $C_{coherent\ quantum\ waves}$.

It is true, the brain is mainly an information device [19], and the structural information of the brain is a part of its physical structure.

A compact disk is a physical device containing structural information, but it is not a cognitive device, it has not its own information processing.

Such physical objects as compact disks and even computers which are using simple programs (non-artificial-intelligence) might be named non-cognitive structural information categories. It may be observed that the language of categories may be extended to any forms of reality [20].

Kato considers a *second part* of $(C_\alpha)_{\alpha \in \Gamma}$ to be *cognitive categories*, therefore categories with cognitive information processing.

Not only the brain, but also the artificial intelligence, has cognitive activity, and also the intelligent robots.

All these information and cognitive processing activities, always on a physical substrate, have in their categories an interior information dynamics in time (when they are in a structural or structural-phenomenological world).

A purely physical world category has not such an internal information dynamics. This explains why Kato considers the physical world categories C_1 and C_2 are *discrete categories,* that is categories with a structure of objects, without morphisms among objects. Still, he writes [6]:

"We consider that C_1 and C_2 are discrete categories with structures. In this formulation, the physical existence, *i.e.*, the object in C_2, of a conscious entity like a human being is only a 'slice' (or a 'foam' like in Zen) of the product category $(\Pi C_\alpha)_{\alpha \in \Gamma}$. For example, non-organic matter M without non-cognitive functions like non-living things in the usual sense can be considered as a presheaf M such that only nontrivial components of M(U) are in C_1 and C_2."

It may be observed that the same form of using the theory of presheaves for consciousness may be put to use for physical objects. A physical object is also a presheaf on *T* with values in K (evidently C_1, C_2,… as discrete categories), where the values in C_1,C_2,… are real physical objects.

If the deep phenomenological part of every purely physical object [15], [12] is taken into account, every such an object is rather a structural-phenomenological object. And the above elements of the theory of Kato are still reliable.

The only problem remains the reference *T*. In such a case his theory is a *theory of the universe* (of one universe in existence). Perhaps it is not a theory of the entire existence, which has not time and has many universes, each with its own time. For the entire existence it would be more convenient to try a *cronos* [21], which is a rudiment of time, a pre-time. Could we use a *cronos* instead of time, confirming in a way the theory of Kato? Would a combination of Kato's theory with the work [12] and also [7]–[14] be possible? A first intuitive answer is yes.

In [12] the main types of phenomenological categories are defined:

- the phenomenological category of the entire existence $C_{phe!1!}$;
- the phenomenological category of a universe $C_{phe.univ}$;
- the phenomenological category of a mind $C_{phe.m}$;
- the phenomenological category of the Fundamental Consciousness $C_{phe.G}$;
- free phenomenological categories.

When it is the case they are examined with their complementary structural categories.

The most interesting case, letting aside the Fundamnetal Consciousness of Existence, is that of the mind (loosely understood as a living being). For a mind, there is a structural physical part which has a complementary phenomenological part [12]. The structural physical part contains also physical structures which are structural information. This structural information has also a complementary phenomenological part which gives to the mind its special properties of qualia, meaning, intuition, etc.

Then, it may happen that a part of the categories of K in Kato's theory are purely phenomenological, in order for the product $(\Pi C_\alpha)_{\alpha\in\Gamma}$ to accommodate consciousness.

THE PRESHEAVES BETWEEN *T* AND K

All the presheaves between *T* and K form a category of presheaves, noted with $\check{T}$.

Let V be an object in *T*. Let $C_1, C_2, \dots C_\alpha \dots$ be the target categories forming K. A presheaf from V to C_1 is P_1, from V to C_2 is P_2, and, in general, from V to C_α is P_α (Fig. 1).

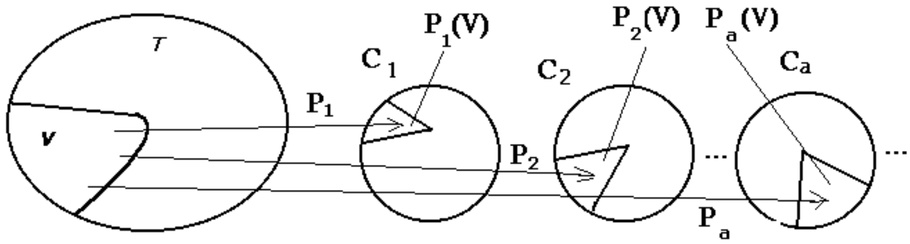


Fig. 1

The presheaf from V to K (Fig. 2) is from V to the product $P_1(V)$ x $P_2(V)$ x...x $P_\alpha(V)$ x ...

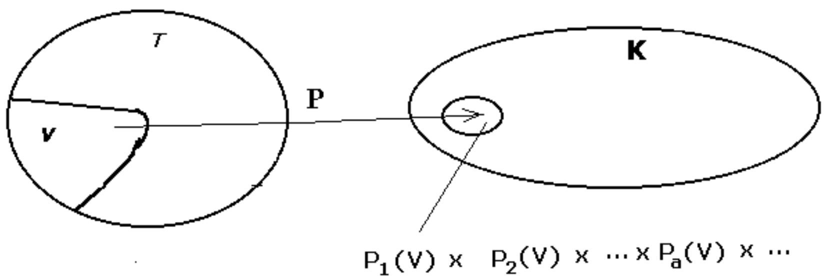


Fig. 2

The object P in $\check{T}$ is a presheaf, in fact a family of presheaves

$$P = (P_\alpha)_{\alpha\in\Gamma} \tag{17}$$

and

$$P(V) = (\Pi P_\alpha(V))_{\alpha\in\Gamma} \tag{18}$$

The category T has many objects (V,W,Y,...) defined in the previous paragraphs. From any such object to $C_1, C_2, ..., C_\alpha, ...$ there are presheaves as shown before.

Therefore the category $\check{T}$ represents all the presheaves from T to K, but because the presheaves are contravariant functors, at every morphism in T corresponds a morphism with inverse arrow in K, or in every C_α. Then it may be written

$$\check{T} : T^{op} \rightarrow K \qquad (19)$$

$$P : T^{op} \rightarrow K \qquad (20)$$

$$P_1 : T^{op} \rightarrow C_1 \qquad (21)$$

A person may use in his life many time intervals (objects in T), another person only a part of the same time intervals, and of course other intervals. For a person, for every time interval there is a sheaf P – formed of $P_1, P_2, ..., P_\alpha, ...$, and the changes from an interval of time to another may be expressed by the morphism $p(v) \rightarrow p(w)$ seen with reference to the corresponding morphism in T^{op}.

A person is

- a collection of objects in T and morphisms among these objects,
- a category of presheaves on his category T,
- and a category of a part of objects of K, which are sections of his category of presheaves, and, of course, the morphisms among these objects.

For all the persons, there is a universe of presheaves (all possible presheaves) on T with values in K.

The expression of Kato (his formula (1) in [5], [6])

$$\check{T} = (\Pi C_\alpha)^{opp}{}_{\alpha \in \Gamma} \qquad (22)$$

may have the meaning that the universe of presheaves $\check{T}$, named by him the conscious universe, comprises the values (the sections) of the presheaves in K, in every of its $C_1, C_2, ..., C_\alpha, ...$, with reference to T^{op} in order to respect the contravariant character of $\check{T}$. Formula (22) is a very symbolic and expressive expression.

SHEAVES IN THE THEORY OF CONSCIOUSNESS AND IN GENERAL FOR THE EXISTENTIAL ONTOLOGY

A *sheaf A* may be generated by a presheaf A,

$$A = \text{Sheaf}(A) \qquad (23)$$

With the details described,for instance by Bredon [16]. In a direct way [16], [3], a sheaf (of Abelian groups or other mathematical structures like rings) on T is a triple

$$(T, A, \Pi) \qquad (24)$$

where T is a topological space (non-Hausdorf, in general) and A is another topological space in a category with direct infinite products, and

$$\Pi : A \rightarrow T \qquad (25)$$

is a local homeomorphism onto T (i.e. $\Pi(A_x)$ = x as shown in Fig. 3; and where each $A_x = \Pi^{-1}(x)$ for x ε T is the *stalk* of A at x.

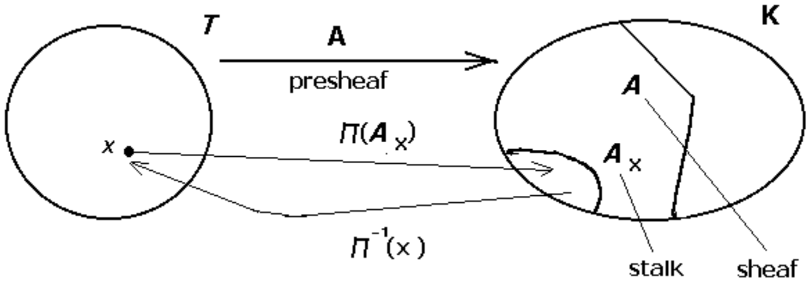


Fig. 3

The sheaf A is a topological space, but it is much more. It has the maps Π and Π^{-1} because they are connected to elements xεT.

A sheaf is not only a topological space in K because it involves the category T with its topological space and the maps Π and Π^{-1}. More, the stalk A_x implies the presheaf. A_x is the set of the germs [16] of the presheaf A about x (about x means open neighborhood of x) and technically [16]

$$A_x = \underline{\lim}_{>} A(U) \qquad (26)$$

where $\underline{\lim}_{>}$ means direct limit and U ranges over the open neighborhood of xεT.

A is the disjoint union of all A_x and "provides information about the local structure of the (presheaf, our note M.D.) A, but most global structure has been lost, since we have discarded all relationships between the A_x for x varying" [16].

To retrieve a global structure a topology of A was introduced.

The topological space < A, topology of A > is the *sheaf generated by the presheaf* A as shown in formula (23).

The presheaf, it is known, is a functor. The *sheaf* implies in its construction a *presheaf.* That is why some authors [17] considers that a sheaf is a presheaf satisfying some conditions.The sheaf may be consquently seen as a preshief functor with added properties. In [3] it is mentioned that these added properties are 'completeness properties', described later down in this chapter.

Perhaps, it would be better to write that a sheaf (a sheaf on T) is

$$(T, \mathrm{A}, A, \Pi, \Pi^{-1}, \mathrm{K}) \qquad (27)$$

implying two categories (of departure and target), a presheaf (which is a functor), a topological space in the target category, and the maps *Π* and $Π^{-1}$.

The section of a presheaf A,from *T* to K is formed by the *objects* of *A*(U) where U is an object in *T*.

The section of a sheaf, named [16] also cross section of *A* over U is a *map* s: *T* → *A* or, more specified,

$$s : Y \rightarrow Z \tag{28}$$

such that (Fig. 4) *Π*.s = 1 (identity),

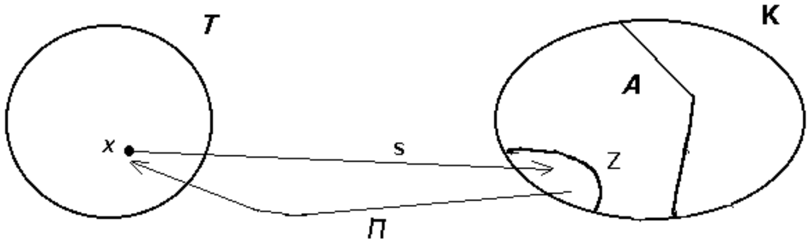
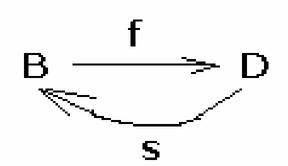


Fig. 4

where *Π* : *A* → Y or, more specified, *Π* : Z → Y. It may be observed the difference between Fig.3 and Fig. 4. In general [22], for a situation between two objects B and D, like that shown in Fig. 5, s is a section of f : B → D if f.s = 1_D.

Fig. 5



As may be seen from Fig. 3, $Π^{-1}$(x) is a stalk for the sheaf (shortly noted *A*):

"This construction (of a sheaf, our note M.D.) *shows a sheaf as a collection of localized stalks* and explains the terminology 'sheaf' for it" [3].

Both stalks and sections are, when the sets in such a theory are Abelian groups, also Abelian groups. But other mathematical structures may be used, as well, for *all* the sets instead of Abelian Groups.

Other two important notions are are restriction of a sheaf and covering of an object in a category.

A restriction [16] is a map in A

$$\lambda_U^V : A(V) \rightarrow A(U) \tag{29}$$

Where A is a presheaf, and U c V in *T*.

If tεA(V) then (29) is *the restriction of* t *to* U and is written t|U. In this case the restriction refers to a section t over U.

Because s= A(U) and s'= A(V) are *thoughts*, being sections of A,the map (29) written as

$$\lambda_U^V : s' \rightarrow s \quad (30)$$

is interpreted [5] as an *understanding* of the thoughts (section) s by the thoughts (section) s'.It may be seen that the thoughts s' are larger (more comprehensive) than thoughts s. Kato observes [5]:

"Thus, brain functions from local information to global information corresponds to realization of the local information as the restriction of the global information in the above sheaf theoretic sense."

When it is impossible to extend s beyond $\check{T}$(V), then s' is said to be a *terminal thought* of s.

Concerning the notion of 'a *covering* of an object in a category', by definition [17] this is a family of morphisms

$$\{\omega_i : U_i \rightarrow U\}_{i\in I} \quad (31)$$

where the range, the object U, is fixed (Fig. 6).

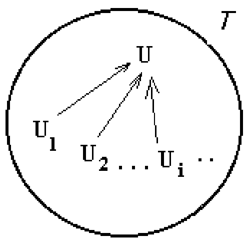


Fig. 6

The set of all coverings of the form (31) is Cov*T*, where in each covering the range U of the morphisms ω_i is fixed.

In a Grothendieck topology, if the objects U and V are open sets, then the morphism U → V, is an inclusion map if U C V , and is empty otherwise (Fig. 7). It may be observed that a Grothendieck topology [17] is based on inclusions.

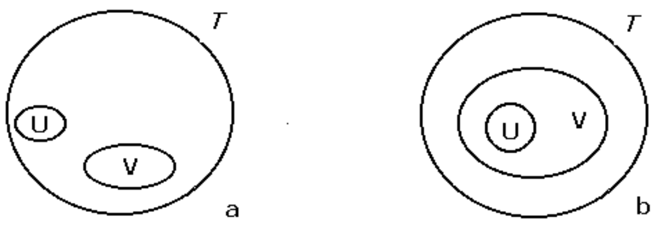


Fig. 7. – Case a is without inclusion, case b with inclusion.

The inclusion morphisms (maps) of a Grothendieck topology are forming a sort of preorder of the objects. Because ω_i,for a Grothendieck topology, are inclusions U_i C U, all U_i being included in U, for such a topology something more is necessary [17], namely

$$(UU_i)_{i\in I} = U \qquad (32)$$

that is, the union of all U_i to be equal to U.

An important consideration refers to the case when a presheaf is a sheaf. A presheaf is a sheaf if the 'completeness properties' are fulfilled [3]. Following [3], let V an open set in *T* and $\{V_i\}$ an open covering of V (knowing $(UV_i)_{i\in I} = V$), respectively, in a Grothendieck topology,

$$\{\omega_i : V_i \to V\}_{i\in I} \in CovT \qquad (33)$$

There are two conditions of completeness properties[3]:

a. If s is a section on V such that all the restrictions to V_i vanish under

$$A(V) \to A(V_i) \text{ for all i} \qquad (34)$$

then sA(V) vanish. This means that 'objects that are locally trivial in A are also globally trivial in A' [3].

b. If there are sections s_i on each V_i

$$S_i \text{ element of } A(V_i) \qquad (35)$$

and the *restricion* of V_i to the intersection of V_i with V_j (*i.e.* $V_i \cap V_j$),which is

$$\lambda_{(Vi\cap Vj)}{}^{Vj} = \lambda_i : A(V_i) \to A(V_i \cap V_j) \qquad (36)$$

coincides with the *restriction* of V_j to the intersection of V_i with V_j, see Fig. 8, which is

$$\lambda_j : A(V_j) \to A(V_i \cap V_j), \qquad (37)$$

for all indices i and j, then there exists a section 's-i'(*i.e.* A(V)) whose *restrictions* to each V_i coincides with λ_i. This means that 'objects which locally belong to A, do actually belong to A' [3].

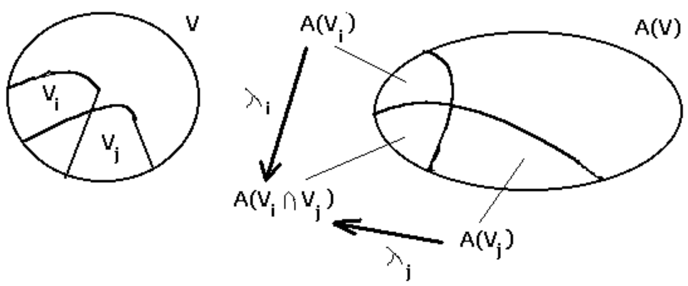


Fig. 8

It has been shown before that a person (having a consciousness) is much more than a presheaf, which is only a functor inside a person. The same is valid for any type of cognitive objects, and also for a physical noncognitive object.

The sheaf described by (27) represents better a person because A gives the local properties of the person and A the integration of these in global properties.

For Kato [5], [6], «a conscious entity, *i.e.* a presheaf in Ť, is said to have thinking ability or coherent understanding ability if the presheaf is a sheaf».

In a sheaf all thoughts are coherent thoughts. That is why a conscious person is better represented by a sheaf.

COHOMOLOGY FOR NETWORKS OF PERSONS (CONSCIOUSNESS)

For Kato [6] a cohomology object represents better "the essence of a conscious entity".

In a presentation of the elements of homological algebra [17] one defines first a complex Σ in a category K to be a sequence of objects with morphisms in chain from one object to the other, the composition of two consecutive morphisms being a zero morphism. Such a complex Σ is itself an object in the category of complexes. Such complexes in K may form a category *Σ*(K).

If to each complex Σ is associated the object

$$Z^n(\Sigma)/B^n(\Sigma) = H^n(K) \qquad (38)$$

that is called the n-th *homology object of the complex* Σ.

For each complex Σ in *Σ*(K) there is a homological object. And the functor from the category of complexes to the correponding homological objects (a subcategory in K) is a *homology functor*.

The key to homology are, in the above formulation, the objects of the form (38). In the theory of categories, the objects of the form (38) are *subquotient objects* [16], [17], [22], [23].

In (38) – see Fig. 9 –,

$$Z^n(\Sigma) = \mathit{ker}\ d_{\Sigma,n} = (\Sigma_{n-1,}\ d_{\Sigma,n-1}) \qquad (39)$$

$$B^n(\Sigma) = \mathit{im}\ d_{\Sigma,n-1} = (\Sigma_{n+1},\ d_{\Sigma,n}) \qquad (40)$$

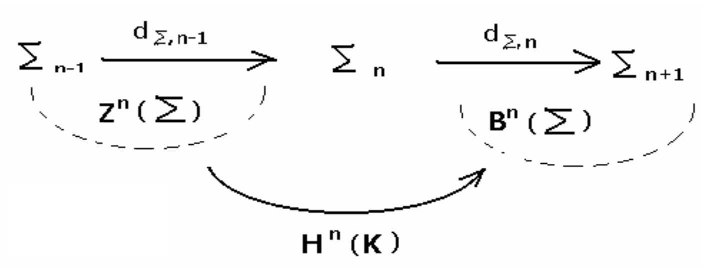


Fig. 9

where (39) is the kernel of the morphism $d_{\Sigma.n}$ and (40) is the image of $d_{\Sigma,n-1}$.

The notion of a *quotient object* is dual to the notion of *a subobject.*

A subobject is a map (inclusion map) from a part of an object to the object (Fig. 10). If we note e: $X_1 \rightarrow X$, the subobject is noted (X_1, e) or even, when exempt of any confusion, shortly X_1.

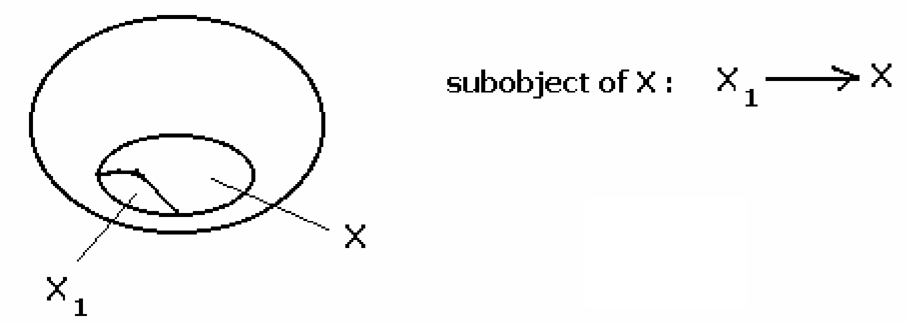


Fig. 10

The subquotient object, being the dual of the subobject,it is the map with inversed arrow. Such a notation is used in (38) and the subquotient object is $H^n(K) : Z^n(\Sigma) \rightarrow B^n(\Sigma) = Z^n(\Sigma)/B^n(\Sigma)$. It may be seen that is a subobject of $Z^n(\Sigma)$, and $B^n(\Sigma)$ c $Z^n(\Sigma)$.

A quotient object may be written, in general, f: $X \rightarrow X_1$,or X/X_1, or, when exempt of any confusion, shortly X_1. The cohomology object is a subquotient object and, when exempt of any confusion, shortly (the n-th component of the entire cohomological object) is called sometimes the cohomology object In such a case $B^n(\Sigma)$ is named also a subquotient object [5].

It may be observed that

$$H^1(K), H^2(K) ,..., H^n(K), ... \qquad (41)$$

being the n-dimensional cohomology object of a complex Σ in K, it is a suit (set of objects) satisfying conditions (38)–(40).

In the category K, which might be the target category in Kato's theory, a complex is positive [17] if

$$\Sigma_n = 0 \text{ for any } n < 0 \qquad (42)$$

In such a case, the complex Σ is named a *cochain complex* and its homology objects (41) are named *cohomology objects* and the homology functor is named *cohomology functor*.

In the homological algebra [17] the cohomology covariant functor is studied mainly, because the contravariant cohomology functor is its dual, and the homology functor, covariant and contravariant, which are dual, are also easily interpreted.

There are also other ways to introduce cohomology theories [17], [16], all being fairly equivalent, even if sometimes with some restrictions. Bredon [16] in a treaty on sheaf theory presents sheaf-theoretic cohomologies, classical theoretical cohomologies (Alexander-Spannier, singular, de Rham, Čech) not based on sheaves, showing they are equivalent, and Borel-Moore cohomology based on sheaf cohomology.

Kato considers that "the true nature of a conscious entity in a complex of network of communication and influence in a society is the cohomological object, *i.e.* the subquotient, not the object itself."

For instance, if two conscious entities P(U) and Q(U'), where P and Q are presheafes of Ť, representing two persons, one related to U and the other to U' in *T*, the sections P(U) and Q(U') are in the category K. Considering the sequence in K,

$$-- - \gamma\rightarrow P(U) - \delta\rightarrow Q(U') - \Phi\rightarrow R(U'') - \eta\rightarrow --- \quad (43)$$

forming a cochain complex Σ (for a complex, two successive compositions are trivial), the meaning of this cochain is of the communications between the thoughts P(U), Q(U'),..., but "the composite of any consecutive communication is trivial" [5].

To a sequence Σ corresponds, as was shown before, a cohomology object, multidimensional, every component of this being a cohomology object related to an object of the sequence Σ. Then the cohomology object at Q(U'), denoted by [5]

$$H^*(\ldots\rightarrow Q(U') \rightarrow \ldots) = \ker\Phi/\mathrm{im}\delta \quad (44)$$

where kerΦ/imδ is the subquotient (see (38)–(40) and Fig. 9) of the object (P(U),δ). The above cohomology object at Q(U') represents the link of Q(U') with P(U) and R(U).

Indeed, the cohomology object is showing the position and links of a person (consciousness) in a network of persons (consciousness). In the frame of his theory Kato is right to use the cohomology theory.

The only problem we see, and to think about, is why the other parts of the constructions are necessary, like *T* and *Ť*, when perhaps it may be possible to work directly with K, with all its structural and phenomenological categories, and where the cohomologies can still be used.

In the case there is only one person (conscious entity) Q(U), the sequence (43) becomes[5],

$$--- \rightarrow 0 - \delta\rightarrow Q(U) - \Phi\rightarrow 0 \rightarrow --- \quad (45)$$

and the cohomology object of Q(U) is Q(U) because no one influences Q(U). Indeed the subobject of Q(U) is the whole Q(U) – there is no other subobject of Q(U). And the subquotient of Q(U) is the whole Q(U) and kerΦ/imδ, which is the cohomology of Q(U),is Q(U) itself.

In the case there are only two conscious entities P(U) and Q(U') the sequence (43) becomes,

$$\text{---} \rightarrow 0 \rightarrow P(U) - \delta \rightarrow Q(U') \rightarrow 0 \rightarrow \text{---} \qquad (46)$$

One observes that no other person (consciousness) may influence P(U). P(U) listens to nothing but has influence on Q(U'). The position of Q(U') shows that it is influenced by P(U) but does not influence anybody.

In a cochain complex (43), because two successive morphisms give a zero morphism (Fig. 9), there is an influence only from one object to the following object and no further (*i.e.* the influence of influence is lost or an influence does not propagate in a cochain complex).

If one takes into consideration a sequence in which a influence propagates, then

$$\text{--} - \gamma \rightarrow P(U) - \delta \rightarrow Q(U') - \Phi \rightarrow R(U'') - \eta \rightarrow \text{---} \qquad (47)$$

is no more a cochain complex and the compositions ...,$\delta.\gamma$, $\Phi.\delta$, $\eta.\Phi$,... are no more zero morphisms,

$$\delta.\gamma \neq 0,\ \Phi.\delta \neq 0,\ \eta.\Phi \neq 0 \qquad (48)$$

In such a case a sequence of quotient (named also subquotient) objects may be constructed, sequence that is a cochain complex [5].These quotient objects are (Fig. 11) Q(U')/im $\delta.\gamma$, R(U")/im $\Phi.\delta$,...

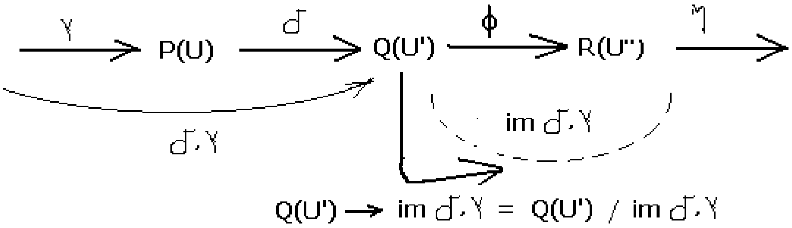


Fig. 11

The cochain complex resulted is presented in Fig. 12.

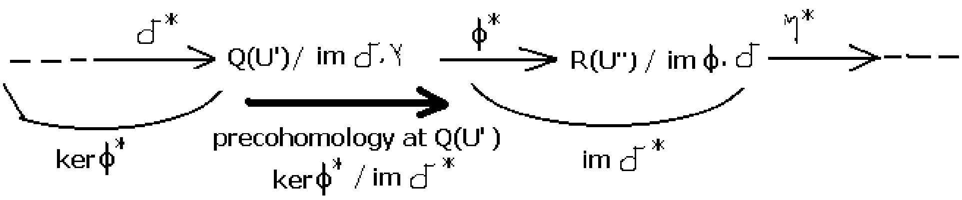


Fig. 12

The *precohomology* at Q(U') is defined as the cohomology of the cochain complex shown in Fig. 12.

The precohomology at Q(U') is noted [5]

$$Ph^*(\text{--}\rightarrow Q(U') \rightarrow\text{--}) = \ker\Phi^*/\mathrm{im}\,\delta^* \qquad (49)$$

Kato [5] observes that the cohomology (precohomology) object, *i.e.* the subquotient not the object itself, shows "the true nature of a conscious entity in a complex of network of communication and influence in a society". Indeed, this might be an important point for the proposed integrative science [18], for the time being at a philosophical level, which is now extended to comprise group and social processes besides structural and phenomenological phenomena [24].

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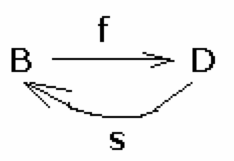
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