

Conceptual and Contextual Hypostases of Abstract Entities¹ Solomon Marcus*

Hypostases conceptuelles et hypostases contextuelles des entités abstraites

Résumé : L'hypostase conceptuelle repose sur des définitions, tandis que l'hypostase contextuelle repose sur des contextes. La définition peut être descriptive ou opérationnelle, tandis que le contexte peut être linguistique ou extra-linguistique. Les définitions descriptives peuvent être axiomatiques ou circulaires, les définitions opérationnelles peuvent être complètement ou partiellement opérationnelles. Les concepts sont le résultat d'une combinaison entre la méthode descriptive et la méthode opérationnelle, mais il y a aussi des définitions purement descriptives (dans certains systèmes axiomatiques) et des définitions purement opérationnelles (dans les différents types de systèmes formels). Le milieu optimum des hypostases conceptuelles se trouve dans les sciences déductives, tandis que le milieu optimum des hypostases contextuelles, se trouve dans le langage poétique.

In the following we intend to make some remarks concerning the ways of acquiring conceptual and nonconceptual significations in the deductive sciences and, in general, within the frame of deductive models which are used in both natural and social sciences.

Significations are introduced either by means of *definitions* or by means of *contexts*. The way of definition is more *systematic*, that of context is more *intuitive*.

Definitions may be classified as *descriptive* and *operational*. Here the word "operational" has nothing to do with the acception promoted by P. W. Bridgman, which concerns an experimental physical operation. A descriptive definition has an internal character, it is concerned with the intrinsic features of the object we intend to define. An operational definition has an external character, it is related to the way in which we operate with the considered object or concept. Operational definitions are more and more used in various fields of research, as a result of the development of the contextual point of view (this fact must be related, among others, to the development of algebra of free semigroups and of the syntactic point of view in various fields like Genetics, Anthropology and Theory of Literature) and of studying of various types of generative systems, beginning with Turing machines, finite automata and combinatorial systems and ending with the general theory of systems, which is now a distinct field of the science. Descriptive definitions are registered in dictionaries. There are two types of dictionaries: usual unilingual dictionaries, where definitions have a *circular character* (see the proof of this fact in Miron Nicolescu [2]), and *axiomatic dictionaries*, i.e. dictionaries where some terms are considered as given (they have a primitive meaning), i.e. they are not descriptively defined, but all other terms are descriptively defined by means of the primitive terms. Despite of their circularity, usual unilingual dictionaries are very useful; this situation derives from the fact that such dictionaries complete their information by means of the contexts. The contextual information is involved here in two ways.

On the one hand the terms we are looking for in the dictionary are already met within at least one context; on the other hand the dictionary explanation involves in most cases some contexts as examples. Thus, circular dictionaries are a compromise, an intermediate position between the axiomatic dictionary and the purely contextual information. Hilbert's axiomatization of Geometry and Kolmogorov's axiomatization of the Theory of probabilities offer examples of axiomatic dictionaries.

Operational definitions may be classified as purely operational definitions and

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partially operational, definitions. A purely operational definition gives a finite number of types of contexts where the considered object may appear, in such a way that none of these contextual situations can be associated with a descriptive definition. If at least one of these contextual situations can be translated into a descriptive definition, then we have to do with a partially operational definition.

Axioms in a formal system are an example of objects introduced in a purely operational way; the only definitional elements are here the various positions, the various contexts in which axioms appear in the proofs associated to the given formal system. None of these contextual situations can be characterized by a descriptive definition. A similar situation have the nonterminal symbols in a formal grammar; they are completely characterized by their positions in the generative rules and in the derivations of wellformed strings and none of these contextual situations is completed with a descriptive definition of the considered auxiliary symbol.

Some appearances are against our assertion. Indeed, auxiliary symbols in a grammar are very often represented by initials of some words designating some grammatical (morphological or syntactical) categories such as noun phrase, verb phrase, sentence, transitive verb, adjective, etc. This situation could give the impression that auxiliary symbols are susceptible of a descriptive definition. Actually, only a notation is involved here, in order to favour the intuition of correspondence between some grammatical categories on the one hand and some auxiliary symbols on the other hand, i. e. the intuition of a relation between some elements which are defined descriptively and others, which are defined operationally. But, in a purely deductive investigation of generative grammars, this relation plays no role.

A very convincing example of purely operational definitions is that given by Hans Freudenthal in his book about a cosmic language [1]. Some basic relations like equality and inequality are here introduced in a purely contextual way, by means of contexts like $---=---$, $---=---$, $----=----$, for equality and by means of contexts like $-<--$, $-<---$, $--<---$, $---<---$, for inequality. In a language for cosmic intercourse no metalanguage can be used, thus there is no place for descriptive definitions. We are concerned here with the dependence of descriptive definitions with respect to a metalanguage and with the possibilities of avoiding the metalanguage by exploitation of contextual capacities of the language. Another problem appearing here is that of the dependence of semantics with respect to the syntax. This problem has been almost completely neglected, in contrast with the inverse problem, that of the influence of semantics on the syntax, a problem to which much attention has been paid.

Examples of partially operational definitions are those given to the number zero in Arithmetics and to some morphemes (such as the morpheme *s* in English) in Descriptive Linguistics. The symbol 0 (zero) is defined in a contextual way, but for almost every type of contexts in which it can appear one associates a descriptive definition. Thus, in the empty context the symbol 0 designates the absence of any unit, whereas in the context 103 it designates the absence of tens. In the context $r + 0 = r$ (r being a real number) the symbol 0 has a purely operational meaning, whereas in the expression 03 of a telephone number 0 is only a logical sign and the same happens in Boolean analysis. Similar situations occur in the behaviour of logical operators of conjunction and disjunction of the propositional calculus and in the definition of the empty set in the Set Theory. In the empty context, the empty set admits a descriptive definition, that of a set which contains no element, but the fact that $A \cup 0 = A$ for any set A involves exclusively a purely operational definition of the empty set. We have here an example of equivalence between a descriptive and an operational definition. The meaning of the English morpheme *s* changes with the context, but to each type of contexts corresponds a descriptive definition of this morpheme; for instance, after a verbal root this morpheme means the third person of the singular, whereas after a noun root it has the meaning of plural.

Most of the concepts used in the everyday language have a partially operational definition. As we have already seen, axiomatic descriptive definitions play an important role in all deductive sciences. Of course, the distinction between descriptive and operational is to some extent a matter of convention, of expression, because we always can legislate some operational, contextual behaviour in the form of some attributes of the considered object, by transforming these behaviours into some internal features. Thus, the operator of logical conjunction (which is a typical operational concept) can be defined in a descriptive way as a binary operator which associates to each pair (p, q) of propositions the value 1 if both p and q are true and the value 0 in the other cases. We can conceive a hierarchy analogous to the theory of types, a hierarchy where the internal features at some level proceed from some operational behaviours at the preceding level.

Another problem is that of the existence and finding of some general rules to pass from an operational definition to an equivalent descriptive definition and conversely. For instance, let us suppose that we have a text whose initial part consists of some descriptive definitions and the remainder of the text consists of some considerations and results about these definitions. Let us suppose that, by some chance, we are unable to consult the initial part of the text, containing the descriptive definitions; does there exist the possibility of reconstituting these definitions by using the remainder of the text? If the answer is affirmative, which is the smallest part of the remainder of the text, necessary to reconstitute the unknown definitions? The length of this smallest part could be interpreted as a measure of the contextual character of the considered concepts, as a measure of the possibility to reconstitute a descriptive definition starting from the manner in which we operate with the corresponding concept.

Sometimes, there is a tendency to ignore operational definitions, by restricting the status of definition only to descriptive definitions. This situation can be seen especially in textbooks. Concepts which are essentially operational are obliged to enter the Procustian bed of a descriptive definition. The number 0 (zero) is such an example. Instead of being introduced operationally, as the null element in the operation of addition (in order to give the whole generality to some operations), it is introduced descriptively, by making no difference between 0 on the one hand and 1, 2, 3, ... on the other hand, thus ignoring the difference of nature between them. Thus, no wonder if a lot of pupils claim that the result of the division of 5 by 0 is 5; they are faithful to the descriptive definition of 0 (the only thing they can find in their textbooks) and they think that if you have five apples and you give them to nobody, then you keep all five apples. Similar mistakes occur in the definition of the empty set, where the descriptive definition "the empty set is the set which contains no element" ("if there is no element, then there is no set" claim with justification some pupils) is preferred to the operational definition.

Such mistakes can be observed not only at the level of the school pedagogy, but at the level of research too. An important French linguist, Bernard Pottier, reproached the generative theory of language for the fact that it uses the grammatical categories without defining them, in other words it uses grammatical categories with their meaning in traditional linguistics [3]. Actually, grammatical categories are very rigorously defined in the generative theory, but not in a descriptive way; they are represented by auxiliary symbols and defined, as we have already seen, in a purely operational way.

Sometimes, an inverse exaggeration occurs, that of underestimation of descriptive definitions. This situation occurs especially in those fields where the operational point of view has been introduced only in recent times and it provokes a disproportioned enthusiasm. Thus, in Linguistics Noam Chomsky and Nicolas Euwet are victims of this exaggeration. They use the pejorative term of Taxonomic Linguistics in order to name that part of Linguistics which is not essentially based on operational definitions. Thus they introduce the false opposition between taxonomic and creative and many linguists imitate them in this mistake.

Of course, there is no superiority of descriptive definitions with respect to operational definitions or conversely. The descriptive and the operational points of view are equally necessary, they complete each other, they allow us to penetrate both the anatomy and the physiology, both the content and the dynamics, both the internal and the external behaviour of the concepts we are studying.

If the deductive sciences introduce their meanings especially by means of definitions, the everyday language prefers the contextual way. We need here an explanation, because the context occurs essentially in some definitions too, it is especially used in operational definitions. But an operational definition indicates exhaustively the types of contexts associated with the considered concept, whereas the contextual way, as a way opposed to the definition, is purely inductive, the list of possible contexts is here permanently open, because otherwise the contextual way is converted into a definition, i.e. into a concept (to close the list of possible contexts means to put a definition by means of which we introduce a concept). Actually, this is just the genesis of many definitions, this is the way in which children acquire meanings.

The contextual way can be of two types, because the context can be of two types: linguistic and extralinguistic. The clearest example of a linguistic contextual way is given by the poetic language. This assertion must be understood in the following way: the poetic language is essentially anticonceptual. The anticonceptual tendency of the poetic language makes it impossible to close the list of possible contexts proposed by the contextual way, practically this list is here infinite.

If the poetic meanings were exclusively the result of contextual situations and if the scientific meanings followed exclusively from definitions, the problem of relations between definitions and contextual meanings would be relatively simple. Actually, we have here only two tendencies. The poetic language tends to elude the concepts, but it never succeeds in realizing this completely; the scientific language tends to crystallize its meanings in the form of definitions, but it always remains tributary to some contextual residues. Take any mathematical definition and you will find some contexts which bring, with respect to the object of this definition, some new information, a new aspect of its behavior, which does not follow directly from the definition. The reason for this situation follows, on the one hand, from the fact that some still unexplicit consequences of a definition appear as contextual surprises, on the other hand from the action of those who use the mathematical language and provoke contextual behaviours which are not contained in the definitions.

This phenomenon may be observed in Elementary Mathematics too. Thus, it is impossible to understand the complexity of the contextual behaviour of the imaginary symbol i if only its definition as square root of the negative unit is used.

Just here we find the origin of some paradoxical statements about the behaviour of i , which allow us, for example, to “prove” that -1 is equal to 1 :

$(-1 = i^2 = ((-1)^{1/2})^2 = ((-1)^2)^{1/2} = 1^{1/2} = +1)$. We can say that by contexts, i.e. by use, we act on the concepts and we modify them slowly, almost imperceptibly, but by accumulation of these small modifications some important semantic changes may be obtained. This phenomenon leads sometimes to spectacular results, which modify essentially the structure of a formal language. Thus, the programming language ALGOL, initially completely conceptualized, fell little by little under the anticonceptual influence of contextual meanings introduced by the programmers. In this way, ALGOL considered initially a context free language (see, for instance the well-known book by Seymour Ginsburg about context free languages) became a context sensitive language. Thus, a distinction which was very clear for everybody became very obscure. How to distinguish a natural language from an artificial language if the latter acquires by usage the features of the former?

To introduce concepts by definitions is somewhat paradoxical. There is a contradiction

between how they are found and where they are placed. Usually, they are found a posteriori, but they are placed at the beginning of the exposition. This is a contradiction between the genetic order and the systematic order of things. The scientific concepts come step by step from contextual meanings and behaviours, but in a systematic exposition we begin with the definitions and we proceed further by discussing contextual behaviours. This is the usual order in mathematical textbooks: first the theory and afterwards applications and exercises. The concept of an integral begins its history in antiquity, the concept of a derivative is introduced only in the 17th century, the concept of continuity is introduced later, at the beginning of the 19th century, whereas the concept of function, in its general form, is known only at the middle of the 19th century. But in almost any textbook of Mathematical Analysis their systematic order is just the opposite of the genetic order: function, continuity, derivative, integral.

We have tried to put in evidence two hypostases of the abstract entities occurring in the frame of deductive models: the conceptual hypostasis, based on definitions, and the nonconceptual hypostasis, based on contexts. There are two forms of the conceptual hypostasis: that based on a descriptive definition and that based on an operational definition. There are two forms of the nonconceptual (contextual) hypostasis: the first is related to a linguistic context, the second is related to an extralinguistic context. Descriptive definitions may be axiomatic or circular, whereas operational definitions may be totally operational or partially operational. Most of the concepts are the result of a combination between the operational way and the descriptive way, but there exist purely descriptive concepts (especially in some theories with a high degree of axiomatization) and there exist purely operational concepts too (like some of those used in the frame of various types of formal systems). Abstract entities which are not of a purely conceptual character, frequent in some deductive sciences, are a source of doubt about the difference of nature between artificial and natural languages. These nonconceptual entities are step by step converted into concepts, but at the same time the use of already existing concepts generates new nonconceptual abstract entities. But the surroundings where nonconceptual abstract entities are at home (i.e. without being dominated by the tendency to be converted into concepts) are beyond science; *they* belong to the poetic hypostasis of the language.

References

1. FREUDENTHAL, HANS : *Lincos. Design for a language for cosmic intercourse*. North Holland Publishing Company, Amsterdam, 1960.
2. NICOLESCU, MIRON: *Problème du dictionnaire axiomatique*. “Cahiers de linguistique théorique et appliquée”, vol. 5, 1968, p. 173-176.
3. POTTIER, BERNARD : *La grammaire générative et la linguistique*. Travaux de linguistique et de littérature publiés par le Centre de philologie et de littérature romanes de L’Université de Strasbourg, vol. 6 (1968), no. 1, p. 7-26.