THE CRITIQUE OF FORMAL DECISION
– ARE DECISION METHODS REALLY METHODS OF DECISION? –

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The issue of decidability is paramount for the analysis of the axiomatic systems of science. In order to decide which formulae function as laws inside an axiomatic system, certain criteria are necessary, based on which the formulae functioning as laws can be distinguished from the other formulae of the system. In modern logic, the criteria of decidability materialise into decision methods: the truth tables method, the normal forms method, the reduction method, etc. One of the most strongly contested aspects related to decision methods is their artificial nature and the fact that they cannot cover the diversity of argumentative practices. Decidability is also criticised because decision methods are far too diversified and also redundant, based on the same principles. Are decision methods really methods of decision? Some methods do not decide, but explain a rational course that cannot be understood at first sight. Strictly speaking, these are not methods of decision any longer.

The issue of decidability is paramount in any logical system: there is no such system that could be comfortable with the rationality of its construction if not able to put order in the corpus of well formed formulae (wff) which can be constructed with its ingredients. But how can this order be achieved? Obviously, by making the distinction between the formulae functioning as laws of the system (tautological formulae) and those that do not function as laws (synthetic formulae and contradictions).

1. THE EXTENT AND THE PURPOSE OF DECIDABILITY

For that reason any constructed system of logic tries to point out methods through which one can have access to the systematization of the domain with regard to the order mentioned above: the distinction between the formulae which function as laws and those which do not. One should not infer from this that the issue of decidability is discussed only by the systems of theoretical knowledge (such as logic or mathematics). This issue is a result of the ambition of pure thinking to rise, in its theoretical constructions, to the highest standards of rationality, where its lawful character and its discovery are undoubtedly some of the key aims. It is to be traced in activities and domains related to the everyday praxis, far from the pure theoretical constructions. And this is irrespective of the fact that the form and the manifestations under which the issue of decidability is traced down are far from being as clearly cut and imperative as they are in the above-mentioned theoretical constructions.

For instance, the legal system of any given society should be conceived in such a way as to allow at any moment *the possibility to decide* on any act submitted to its analysis. On the basis of the legal system, any act committed by an individual – if it is one of the acts "regulated" by the legal system – must be decided upon in one way or another, meaning that a verdict should be given on the act of that individual. Even if in this case the issue of decidability is not the same as in the theoretical constructions of logic or mathematics, we still have to admit that the continuous improvement of legal systems aims exactly at a more adequate functioning of the imperative of decidability: we improve laws in order to decide more easily and to reduce the possibility of error making! This idea also applies to the field of moral relations: we judge and often punish, morally speaking, other people's deeds because the system of moral norms of a group or a community gives the possibility to decide whether a deed is right or wrong.

These are some examples that lead us to the assumption that the issue of decidability is a general one for human knowledge. Therefore it is all the more pressing for scientific knowledge and for some of its fields, where this issue is a key one. That is why argumentation – as a rational practice – needs to know whether the arguments used by speakers are correct or not. If they prove to be correct, then the results of such an approach prove an adequate understanding of the world; if they do not, we face an apparent understanding that can do more harm than the lack of understanding. A series of methods developed by modern logic [1] have been suggested as criteria for delimitating correct reasoning from incorrect reasoning (source of error) in an argumentation.

While preserving the critical intention from the very title of this paper and also aiming at answering the question by which we opened the discussion on decidability, we intend to point out some aspects that question the limits of decidability in human knowledge. However, we have no intention to conclude that we cannot trust the decision methods concerning the argumentation techniques we have emphasized.

2. THE DECIDABILITY AND THE CRITERION OF DECIDABILITY

Therefore, the issue of decidability is a general issue of human knowledge. Whatever the field, there must be a possibility to determine whether a given construction specific to that field is under the sign of truth and correctness, or under the sign of falsity and error. But the fact that the issue itself is general does not make the decision methods universal, that is, appropriate for all the fields of human knowledge. **The definition** of decidability is universal: to identify the truths in any field of knowledge as against the wrong statements. **The criteria** of decidability are different for each field of knowledge where we try to identify them in practice. This situation is similar to the one pointed out by Alfred Tarsky in his famous work *The Concept of Truth in Formalized Languages*: the definition of truth is one and the same in all fields, only the criterion that materializes this definition into the practice of knowledge differs from one field to the other [2:152–278].

Discussions upon and analyses of the argumentation techniques and of the methods of determining their formal correctness support us in suggesting this distinction. The issue of decidability has the same framework both at the level of inferential deduction techniques and at the level of syllogistic deduction techniques: that is, the imperative to determine whether the argumentation techniques, materialized in various types of reasoning, observe the correctness rules of rationality. In order to fulfill this mandatory imperative, some methods are necessary that function as discriminative criteria and by means of which we can prove if a reasoning (in its quality of argumentation technique) is right or wrong.

We can note that these methods – these decidability criteria – differ from one logical system to another. We use some methods for determining the validity of inferential deduction techniques (the truth tables method, the semantic graphs method), and other methods for the syllogistic deduction techniques (the reduction method, the diagram method). It is maybe necessary to make here a statement that could eventually soften the reproaches. It is true that there are also, at least by name, decision methods common to all techniques (the *reductio ad absurdum* method, the natural deduction method). In other words, considering the methods in a too tight relation to the cognitive system is not always justified. We want to point out that the transfer and functionality of a method from one field to another, when possible, are always accompanied by new elements and rules that can make the method applicable under the circumstances of the new field. For instance, in the case of the inferential deduction, the *reductio ad absurdum* method starts from the assumption of the falsity of the formula and ends with discovering the contradiction, whereas the same method applied for testing the syllogistic techniques requires more complex intermediary constructions in order to reach the same result. On the another hand, the natural deduction method, common to the two systems of argumentative techniques, requires new rules (*e.g.*, rules of adding and removing quantors) in order to become functional in validating syllogistic techniques, which is not the case for the inferrential techniques.

Where does the imperative of sorting by fields the validity determination methods come from? All in all, where does the actual functioning of decidability come from? It comes from the fact that the method, in its quality as a discriminative criterion, belongs to that field of knowledge where the principle or the need of decidability functions. Decidability concerns the sentences of one field exclusively, and the nature of the sentences of a given field can only be defined by means of “instruments of measurement” able to take into account and to reflect the specific characteristics of that field. The decision methods are such instruments of measurement. Actually, the issue of the applicability of the measurement instruments to the specificity of the object to be measured can also be approached in a broader sense, which confirms that: we can not measure psychic phenomena with mathematical methods (numerous errors have resulted from such an illicit transfer of method to a field where it was inappropriate), and we can not evaluate economic phenomena with methods from chemistry!

At this point, we are in the middle of a *methodological antinomy* [3]: the ideal of decidability, at least in scientific knowledge, tends towards universality (there has always been an obsession for finding decision methods as widely applicable as possible), while in practice this ideal has always been reached by methods as close as possible to the field of application and its specificity. The methods’ tendency to universality results in losing (blurring) the field, while the tendency to applicability results in losing the explanatory amplitude.

3. THE ARTIFICIALITY OF FORMAL DECISION

Much criticism on formal decision incriminates the fact that methods, as they are presented, seem to have an obvious **artificial nature**, being far from covering the diversity of argumentative practice where the argumentation techniques are applied [4]. They seem rather like a game of mind able to control what should ideally occur in the practice of argumentation, but they become quite vulnerable when it comes to real usual critical disputes. Could it be possible to apply these methods in a usual dispute with an interlocutor if we find that he or she uses an incorrect argumentation technique? Difficult to uphold, the skeptics would say. Although it is not in our intention to transform the skeptics into optimists, we do want to point out some aspects that could soften the mistrust of those confronted for the first time with such an evaluation of argumentation.

For example, if we adopt the matrix method we must first point out the fact that determining the possible combinations of truth-values of the elementary propositions that make up an argumentation technique (the first step in method activation) means, in fact, **interpreting** the propositions in terms of truth-values. Actually, the truth-value of the reasons is essential for an argumentation; in fact, we are always concerned if the propositions we use as reasons are true or false, and if the interlocutor’s counter-argumentation is based on reasons expressed by true or false propositions. This is due to the fact that, in both situations, the functions of argumentation are different: if the reasons are true propositions, then the thesis can be sustained; if they are false, then the disputant rejects them, and this way the thesis is also rejected.

Secondly, we would like to point out that each interpretation of elementary propositions (which generates an interpretation of the formula or of the argumentation technique) is *a possible instantiation of the argumentation technique in the practice of argumentation*, that we analyze for all possible interpretations. In the practice of argumentation, an argumentation technique is used in one or the other of the possible interpretations given to its structural components – the elementary propositions. If the technique is correct, then each one of its instantiations is correct and therefore the argumentation is correct. For example, the argumentation

> Water froze *because* the temperature is below zero

is the short form of the reasoning

> If the temperature is below zero, then water freezes
> But the temperature is below zero
> Therefore: Water froze

which can be formalized as

$$[(p \rightarrow q) \,\&\, p] \rightarrow q$$

Using the truth tables method we can decide that this reasoning is the expression of a logical law:

$[(p \rightarrow q) \,\&\, p] \rightarrow q$
T T T T T T T
T F F F T T F
F T T F F T T
F T F F F T F

The evaluation method shows that the argumentation technique is valid for all possible interpretations. Actually, many times, without being aware of all these stages, we proceed to determining the interpretation that an actual argumentation assumes in order to see whether it is correct or not.

We used the truth tables method as a “case study” to illustrate the accusation of artificiality brought quite frequently to formal decision, but this is not the only method concerned. The question arises if decomposing the formulae according to rules that look rather like conventions ensuring the practical functionality of the method has actually no significant results in the argumentation practice, when we need to determine the correctness of certain argumentation techniques and we have the ambition to use the semantic graphs method. Or isn’t it true that, in the case of the natural deduction method, the operations applied to the given premises in an argumentation are artificial rules leading to the intended result (because if we don’t reach the intended result, we introduce one more hypothesis!); and that they are not procedures that could be applied to the practice of argumentation and that could reveal something about the correctness of the argumentation?

Without any further consideration on the accusation of artificiality brought to formal decision, we must point out that all the procedures, combinations and operations that we use to explain the logic of our thoughts and that we consider extremely artificial, are the very operations that take place spontaneously in our mind during our usual reasoning. The fact that these operations take place fragmentarily, that some stages are often skipped, that they are covered by exterior considerations, does not justify our ignoring the connection between the theoretical explanatory approach and the actual practice of thought.

The artificiality has consequences on the practical functioning and productivity of some of the decision methods. Going back to our case study, we would like to point out that the truth tables method is a useful instrument for determining the correctness of an argumentation technique on the condition that the number of elementary propositions in the argumentation is relatively small (2, 3, 4). For an argumentation technique with two elementary propositions, we shall use four combinations of the truth-values; for three elementary propositions, we shall have eight combinations; whereas for four elementary propositions, we shall reach a number of sixteen combinations of the truth-values. If we have six elementary propositions, the number of combinations is already irritating: 64. But what if the number of elementary propositions is even bigger? The inconvenience of this method is not one of a logical-formal nature: the method can function and can lead to the decision irrespective of the number of elementary propositions and of the combinations they can generate. The difficulties arise at the operational level and with regard to the efficiency of the method: with so many combinations to be determined, the method is difficult to apply and often tiresome! As tiresome as other methods when they are used to differentiate, in the multitude of reasonings of the discursive practice, what is right from what is wrong.

4. THE REDUNDANCY OF DECISION METHODS

What seems to be no longer a surprise for an experienced analyst, but amazes the novice that comes into contact with these aspects of knowledge, is this endless **multiplication** of the criteria (the methods) used to delimitate the truth from the error, those gate keepers nobody can pass without satisfying the minimal requirements of rationality.

This methodological multiplication takes place in many directions. First, there is a multiplication of decision procedures according to the field where they are used to put into practice the requirement of decidability. As stated above, methods are related first to their field, so the task of the latter was to find and explain the methods that are the most productive in reaching the purpose for which they were meant. In the two fields of traditional logic (the only ones that use reasoning) we can find decision methods in the logic of complex propositions. Some of them date long back in the history of the discipline ("reductio ad absurdum"), others are more recent (the matrix method), as decision methods appeared in syllogistics, too, some of them having an age-old tradition (the method of reduction can be found in Aristotle's work), others resulting from modern times contributions (the diagram method, the natural deduction method).

At a second stage, the multiplication of criteria takes place in each field separately. Each system of logic, by its most authorized representatives, deems right to release on the market as many decision methods as possible. It has the conviction, not without a reason, that a great number of determination criteria of valid formulae in the system proves the consistency of the system as a rational construction. Either named after the person who conceived the method (Beth's method of semantic tables, Venn's diagram method, Carroll's diagram method, Quine's method) or not (the matrix method, the normal forms method, the semantic graphs method, the antilogism method, the natural deduction method), the number of decision methods has increased and it will probably increase even more in each field, so that the situation seems out of control.

A question arises: is the impressive multiplication of decision methods a virtue of knowledge? Or is it, on the contrary, a limitation of ideal constructions that emphasizes even more the fact that limitations are sometimes unacceptable at the level of pure thought? Let us start with the virtues of such methodological pluralism of decidability. The increasing number of decision methods and their diversification in each field of knowledge is, no doubt, a sign that each field has its specificity and must be analyzed with proper methods. From this point of view, the increase in the number of methods is a means to ensure a knowledge as adequate and realistic as possible of the phenomena of a given field. Either in the case of importing and adapting methods, or in that of creating new methods, this multiplication is a sign of a genuine preoccupation for the accuracy of knowledge.

The multiplication of methods in each field is also a sign of the critical self-consciousness of that field: we need to check our statements several times and, if possible, with different instruments, in order to see if the results are and remain the same. This is because any method, as complete and ingenious as it may be, can have gaps that could lead to errors in knowledge. But when the number of procedures increases, there are fewer chances to leave out certain aspects, because of the complementarity of the methods.

Nevertheless, one cannot overlook the fact that multiplication has several limits, some of them already pointed out by the critics of decidability. One of the most frequently asked questions concerning the diversification of methods for each field of knowledge is: what does this distribution of decidability methods by fields suggest? Is it a reflection of the long insinuated fact that human knowledge is ‘parceled out’ by fields and that humans cannot leave one field and enter another without the fear of making mistakes? Attaching methods to fields seems to suggest an affirmative answer to these questions, even if this assumption is not fully supported by all scientific facts.

On the other hand, having several truth validation methods in each field is doubtlessly a good thing, but isn’t this excess – because excess is what we obtain if we make a realistic inventory of these methods – a pointless ‘trouble’ for anyone trying to check the truth of a statement? Aren’t we here in the situation of a merchant whom a client would ask to weigh his merchandise with dozens of different balances? What is the point of all this effort if the result remains the same in the end?

The idea of excess of methods in different fields of knowledge is also pointed out by the fact that some methods are redundant, that is they can be reduced one to the other or they are founded on the same functional principles that ensure the decision process. For instance, the semantic tables method, as well as the semantic graphs method, can be easily reduced to the *reductio ad absurdum* method because they have the same functional principle: the presupposition that the formula is false leads to contradictions, which are a sign that the presupposition is false. The Venn diagrams method and the Carroll diagrams method also function on the same principle in testing syllogistic techniques: the representation by diagrams of the premises wherefrom the representation of the conclusion must result. There is no doubt that many other examples could be given.

5. ARE DECISION METHODS REALLY METHODS OF DECISION?

But the most difficult question concerning decidability is whether decision methods really ensure, by their functioning, the decision regarding the given reasoning, that is whether they can show that their formulae are logic laws or not.[1] To decide upon the correctness of a reasoning act means to prove that such an act of thinking is, in all possible interpretations, a true formula. Do decision methods reach this purpose? In our opinion, not all of them and not always.

If we have an elementary formula, for example an inferential mode of the type **ponendo-ponens** (which is a usual argumentation technique), and we want to test its correctness by using the matrix method, we obtain the following result:

[1] The issue was discussed by the author at the Seminary of Logic at the University of Neuchâtel (Switzerland) in the autumn of 1996, on the occasion of a debate over some aspects related to figurative decision techniques. Some of the other participants were Jean-Blaise Grize and Denis Miéville (Université de Neuchâtel), James Gasser (Université de Lausanne), Guillaume Martel (Université Laval, Québec).

$$[(p \to q) \;\&\; p] \to q$$

T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F

which shows that the formula is true in all the four possible combinations of the truth-values and that it is therefore a logical law. The argumentation technique that is based on a logical law is correct. We can conclude that the method proved that, irrespective of the alethic interpretation we may give to the elementary propositions, the reasoning in its whole is correct. Therefore, the method provides the decision concerning the truth-value of the formula and also confirms its place in the category of logical laws. This is a case where the decision method seems to serve decidability and it really is what it is supposed to be.

But this does not happen always. If we test the same formula by using the **normal forms method** (which we have not analyzed here, but that is not the point now), we shall proceed systematically to the transformation of the given formula into equivalent formulae until we obtain a conjunction of disjunctions of elementary propositions:

$$[(p \to q) \;\&\; p] \to q \;\equiv$$
$$-[(p \to q) \;\&\; p] \lor q \;\equiv$$
$$[-(p \to q) \lor -p] \lor -q \;\equiv$$
$$[(p \;\&\; -q) \lor -p] \lor q \;\equiv$$
$$(p \lor -p) \;\&\; (-p \lor -q) \;\equiv$$
$$(p \lor -p \lor q) \;\&\; (-p \lor -q \lor q)$$

The last formula is the **normal conjunctive form** of the given formula. In this formula it can be easily noticed that the two disjunctions are true (whatever the truth-value the variables would be given, the disjunctions will always have the value **true**) so, consequently, the conjunction of these disjunctions is a true formula. What has the adopted method done, after all? It has brought the given formula, through equivalent forms, to such a simple form that anybody can see it always has the value **true**! As formulae are equivalent to one another, it results that each of them says the same thing (has the same signification), but the last one says it more explicitly, in a more intuitive way, at the level of comprehension of the receiver. Ideally, a genious mind should be able to "see" the truth in the very first formula, just like an ordinary mind can see it in the last formula.

What is the conclusion of all this? The important conclusion is that the method is not really a decision method, but rather a **method of explanation**, founded on more adequate intuitive bases – an explanation of what cannot be understood by reasoning in the first given formula, which seems much too complicated for the alterity. This does not at all mean that such a method is not important; on the contrary, it is absolutely necessary in order to explain the correctness of argumentation techniques to the alterity. But we would like to draw attention upon an assumption that functions as prejudice in the field: certain methods presented as unfailing criteria of decidability are, at a closer analysis, only but true and important didactic methods able to explain what common sense cannot perceive, even if it is included in the given premises for argumentation.

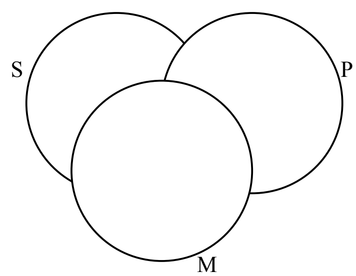
This goes also for the evaluation of syllogistic argumentation techniques. Let us take a **Barbara** mode syllogism:

Writers are imaginative
Poets are writers
So: poets are imaginative.

To test its correctness by means of the Venn diagrams method [5:165–177], we rewrite it in the language of classes:

$$\frac{\begin{array}{l} M-P = 0 \\ S-M = 0 \end{array}}{S-P = 0}$$

then we transfer the information from the premises into the circular diagrams, according to the well-known rules of the method:



We can see that diagramming the premises resulted in diagramming the conclusion. According to the rules of the method, the syllogism is valid and the technique based on it is correct.

It is only here that the discussion on method starts. What have we obtained, in fact, by rewriting the information from the premises (given in natural language) by means of the circular diagrams? In our opinion, nothing else but a **clearer representation** of this information, a representation that facilitates the intuitive understanding of the relations between the notions in the premise-sentences. Does the diagrammatic representation bring more information than what the premises expressed in natural language already contain? Obviously not. It would not even be possible because, according to the rules of the method, all we can do is rewrite the information from the premises into diagrams. Expressing the syllogism in diagram language only brings a more intuitive and adequate explanation of this information. Again, we notice that the method assumes undeserved qualities: it does not really decide, but explains what is said in the premises of a syllogism to those who cannot understand it yet. We repeat here the idea that a genius mind should be able to see in the premises expressed in natural language exactly the way an ordinary mind can see the relations between notions in the diagrammatic representations. However, let us admit that evaluation methods are meant for ordinary minds!

The cases above are not the only ones to be taken as examples. If we test an argumentation technique based upon a reasoning of the type **ponendo-ponens** by means of the natural deduction method, we shall have:

(1)	$[(p\rightarrow q) \,\&\, p \mid\!\!-\, q$	
(2)	$(p\rightarrow q)$	&E (1)
(3)	p	&E (1)
(4)	q	$\rightarrow$E (2 × 3)

where we can see that, by using functor elimination rules, we reach the conclusion (q). This method shows us the steps we take in applying different rules to the given formula or to its resulting formulae. Normally we should be able to see, in the given formula itself, the possibilities it has to reach the conclusion if correct operations are used. Basically, the method explains this itinerary.

6. CONCLUSIONS

Our observations and emphases in the present paper, especially those in the final part, are not meant to destroy common places long rooted in the science of logic. They are only meant to suggest another point of view and, if possible, to lead to discussions and maybe to a reevaluation of the issue of decidability and of its instantiation criteria. On the other hand, these observations also aim at proving that decision methods are open to the alterity, as many of them can only look like more adequate explanations of what is given in the premises of argumentation.

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