AN AXIOMATIC SYSTEM FOR THE LOGIC OF ACCEPTANCE

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1. INTRODUCTION

The first time I have sketched a logical theory of acceptances it was twenty years ago, while trying to describe the dynamics of the cognitive states making use of epistemic automata [31, pp. 447–451]. Our interest for the logic of acceptance was recently brought about by our effort to build up a semiotic, actionalist and epistemic theory of argumentation.

Argumentation takes place in a given action situation and discursive context during a dialog between a speaker or a writer and his addressees. The term argument is an ambiguous one. Currently we can understand by the term a proposition which supports a thesis, the argued thesis and all its propositional supports or an iterated process of inventing arguments and counterarguments for a given initial problem or thesis and all other related and modified theses subjected to a contradictory dispute.

We are in full process of reconsidering our standpoint of view in the argumentation theory expressed in some previous papers or courses [36–40]. The present paper on the logic of acceptances is part of a more extended chapter on this subject included in a new book of ours on the argumentation theory. In this chapter we present a trivalent logical system of acceptances and a modal theory of acceptability and rejectability connected with the notions of assertion, conviction, sincerity and lie. We are also interested in the study of the dynamics of acceptances and rejections making use of the epistemic and doxastic automata, normal forms, Horn clauses and logic programming.

The logic of acceptance is a theory of decision in action contexts described by means of predicate logic referring to states, properties, relations and human actions or abilities. The acceptance of a statement, proposal, offer, request a.s.o. is always intimately connected with the values and goals assumed by the agents, with their social status and roles. The act of acceptance is a kind of local option or decision making of a given agent. Any acceptance supposes a value judgement. Acceptance is a species of practical attitudes of the agents, a kind of metavalue because we accept true assertions, positive or good deeds, adequate means or programs.

What does a human being accept and why? Currently a human being accepts a statement in virtue of its truth, an offer because he or she considers it profitable. An ill man accepts a drug because a physicist has recommended it and because he considers it adequate to cure the disease or complaint which he is suffering from. Men always accept things they consider good, dear, useful, pleasant or beautiful and always reject the opposite ones.

We can formulate different criteria and acceptance rules for each class of agents having a given status or role, placed in a given hierarchical structure or pursuing some assumed goals. From a logical point of view we can express these rules by means of *if, then* statements or by means of Prolog instructions. If we add to these rules adequate factual knowledge bases, then we can predict some agents' value judgements or their positive or negative attitudes.

In order to illustrate the large field of the logic of acceptance, we shall enumerate several species of utterances and speech acts which can be accepted or rejected:

We can accept:

– a statement, because we consider it to be true;
– a solution of a problem, because it is correct and satisfies all the restrictions stipulated on it;
– a proposal, because it is convenient for us;
– an offer to buy something for the same reason or because we are in need for the given object or service;
– a decision because it is the best we can make in a given state of affair;
– a performance because the actors express in a very convincing way the author's intentions and for the high qualities of the actors;
– a scientific paper for its creative content;
– the cooperation with an agent since we are in need for his qualitative service, we have trust in his ability or competence;
– the program of a political party because we appreciate their goals, ideals and strategy and we appreciate the leaders;
– a thesis of an interlocutor because of the truth of his arguments and since the thesis is a logical consequence of his arguments.

Each one of the above species of acceptances can be expressed as an *if, then* statement or as a logical program rules.

Acceptances may be understood as value judgements or as pragmatic-operational attitudes adopted by a human being as a decident agent and founded on his state of knowledge about the actual state of the affair, on his set of explicit or implicit criteria for valuation.

2. INFERENCE SCHEMATA AND LOGIC OF ACCEPTANCE

We intend to propose a weak system of the logic of acceptance on the basis of a K modal system reinterpretation. This system has a single modal axiom founded on a *modus ponens* schema interpreted as an acceptance rule of inference. We shall call this system the AK system of the logic of acceptance. Its axiom says:

Rule A. If someone accepts a conditional statement and if he also accepts the antecedent of the conditional proposition, then he must also accept the consequent of the conditional statement.

$p \supset q$	$A(p \supset q)$	$A(p \supset q) \supset (Ap \supset Aq)$ (A MP)
p	Ap	*Modus Ponens Acceptance Principle*
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q	Aq	

N.B. We can interpret as rules of the logic of acceptance all the five inference schemata proposed by the Stoa and Megaric Schools. Furthermore, we can interpret as logic of acceptance rules all the propositional or predicate schemata. The Principle of Resolution and Splitting Rule or Davis Putnam Theorem can be also interpreted as logic of acceptance principles. Moreover, if we admit the definition of the rejection as the acceptance of the opposite or negated proposition*, i.e.*

1. $Rp = A\text{-}p$,
then we shall accept the inference schemata:

$p \supset q$	$A(p \supset q)$	$A(p \supset q) \supset (Rq \supset Rp)$ (A MT) –
q	Rq	*Modus Tollens Acceptance Principle*
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$-p$	Rp	

To accept an implication or a conditional statement and reject its consequent means to reject its antecedent.

Alternative schemata can be also interpreted as logic of acceptance schemata.

$p \vee q$	$A(p \vee q)$	$(A(p \vee q) \wedge Rp) \supset Aq$ (P Alt)
$-p$	Rp	*Alternative Principle as Rejection Rule*
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q	Aq	

To accept a disjunctive proposition and to reject a term of the disjunctive proposition means to accept the remaining one.

The Conjunction Rejection Schema can be also interpreted as logic of acceptance schemata:

$-(p \wedge q)$	$R(p \wedge q)$	$R(p \wedge q) \supset (Rp \vee Rq)$
---------	-----------	*Conjunction Rejection Schema*
$-p \vee -q$	$Rp \vee Rq$	

To reject a conjunction means to reject one or another of its terms.

The principle of Incompatibility is a variant of the Conjunction Rejection.

$-(p \wedge q)$	$R(p \wedge q)$	$(R(p \wedge q) \wedge Ap) \supset Rq$ (P Incop)
p	Ap	*The principle of Incompatibility*
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$-q$	Rq	

To reject a conjunction and to accept a term of this means to reject the other.

The principle of transitivity can be also interpreted as logic of acceptance principle.

$p \supset q$	$A(p \supset q)$	$(A(p \supset q) \wedge A(q \supset r)) \supset A(p \supset r)$
$q \supset r$	$A(q \supset r)$	Transitivity Principle (P Tranz)
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$p \supset r$	$A(p \supset r)$	

To accept a chain of implications means to accept that the first antecedent implies the last consequent. (In a chain of implications each consequent becomes antecedent in the immediate implication, except for the last consequent.)

3. RESOLUTIVE DERIVATION AND LOGIC OF ACCEPTANCE

Let λ be a literal, C and B propositional disjunctive clauses Then $\lambda \vee C$ and $-\lambda \vee B$ will be disjunctive clauses and $C \vee B$ their resolvent. The Resolution Principle states that if the clauses $\lambda \vee C$ and $-\lambda \vee B$ are both true, then their resolvent will be also true.

$\lambda \vee C$	$A(\lambda \vee C)$	$(A(\lambda \vee C) \wedge A(-\lambda \vee B)) \supset A(C \vee B)$
$-\lambda \vee B$	$A(-\lambda \vee B)$	*Acceptance Resolution Principle*
(P Rez)		
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$C \vee B$	$A(C \vee B)$	

Someone who accepts two clauses which contend a pair of opposite literals must also accept their resolvent or logical imediate cosequence.

The resolution principle is generalised by the theorem of resolutive derivation. Transposed in the logical theory of acceptances this says:

Someone who accepts a given knowledge basis rendered in a disjunctive normal form or as a set of clauses, must also accept all its derived clauses or resolvents.

Especially someone who accepts the rules and the factual basis of a Prolog program must also accept every fomula derived therefrom.

Conversely, if we can reject a derived formula from a a set of KR rules and from a set of factual basis FK, then we must reject at least a fact from FK or a rule from KR. Let us suppose that we have a clause k derived from a KR set of rules and from a FK set of facts which is rejected on the basis of independent and irefutable reasons, *i.e.*:

$A(KR \wedge KF \models k)$	(1)	hypothesis
Rk	(2)	hypothesis

then:

$A(KR \wedge KF \supset k)$,	(3)	(Deduction Theorem, 1)
$R(KR \wedge KF)$	(4)	(Modus Tollens, 2, 3)
$R(KR) \vee R(KF)$	(5)	(Conjunction Rejection Schema)

These principles or schemata of the logic of acceptance will interfer further as inference rules in our logical theory of action and in our new version of a logical theory of argumentation.

4. LOGIC OF ACCEPTANCE AND EPISTEMIC, DOXASTIC, DEONTIC AND TELEO LOGICS

We can construct the logic of acceptance as a generalization of epistemic, doxastic, teleologic and deontic logics.

If something α is known, believed, pursued as goal or obliged for the agent x, then it is accepted by the agent x. I will express the above principles by means of the associeted inference rules:

$$\frac{K(x, \alpha)}{A(x, \alpha)} \qquad \frac{B(x, \alpha)}{A(x, \alpha)} \qquad \frac{S(x, \alpha)}{A(x, \alpha)} \qquad \frac{O(x, \alpha)}{A(x, \alpha)}$$

The first rule says that if something is known as true, then it is accepted. This supposes that the accepting person is moral. The second rule says that we accept the things in which we believe. The third rule says that we accept our goals. The last rule says that we always accept our obligations. This is only true for correct or legalist agents which always observe their obligations and interdictions. This is not the case of the offenders, anarchists, revolutionists and other nonconformists.

We can see the logic of acceptance as a kind of general axiology regulating our value judgements in practical, technical, juridical and moral affairs.

We can connect our logic of acceptance to the theory of qualitative control and technical standards. For each technical process leading to a pursued terminal state or finite product we can define a set of acceptance conditions making use of *if then* statement, Horn clauses or Prolog rules. This way our abstract and speculative theory of value judgements may be connected to different particular cases from different fields of practical activities.

accept(x, α): – cond 1(x), cond 2(x) . . . cond n (x) .

Our first conclusion is that we can see the logic of acceptance as a generalization of epistemic, doxastic, deontic and teleologic systems.

Secondly, the logic of acceptance can be interpreted as a general axiology or as a logic of value judgements.

Thirdly, the logic of acceptance can be connected to several technical, juridical or administrative activities and to the theory of qualitative control and technical standards.

Finally, for each field of application or for each technical proccess their own conditions for acceptance can be associated.

5. THE NORMAL MODAL K SYSTEM AND THE AXIOMATIZATION OF THE LOGIC OF ACCEPTANCE. THE AK SYSTEM

The modal system K supposes the propositional logic or first order predicate logic and the axiom K

$$K(p \supset q) \supset (Kp \supset Kq) \qquad \textbf{(K)}$$

In addition to this infrastructure and axiom K, the system makes use of the Necessitation Rule:

$$\frac{\models \alpha}{\models K\alpha} \text{ i.. e. } \frac{\models \alpha}{\models A\,\alpha} \qquad \text{(AN)}$$

The Necessitation Rule may transform every valid formula of propositional logic or first order predicate logic into a logical law or theorem of a modal logic system. The necessitation rule was proposed by Kurt Godel, the creator of the normal modal systems. This means each valid formula of propositional logic or first order predicate logic can be converted into a formula of a modal system as the alethic modal system, epistemic, deontic, teleologic, a.s.o. Applying the necessitation rule to the epistemic, we come to say that if α is a tautology or a law of the propositioal logic, then this law will be known by the referring agent. I do not think that all logical laws are actualy known by each thinking agent. I do not believe that there is in the world at least a logician who know all the laws of the logic. But I think that all sensible logicians accept that all logical laws of propositional logic or first order predicate logic can be proved by a finite number of steps as valid formulas.

A necessitation rule for the logic of acceptance will be something more natural than the neccessitation rule for epistemic logic.

If $\models \alpha$, then $\models A\,\alpha$ (AN)

If α is law of the logic, then α will be accepted.

Analogous to system K, the system A supposes the laws of propositional logic or of first order predicate logic, the uniform substitution rule, SR, modus ponens, MP, and the extensionality rule, ER, and, in addition, the necessitation rule, N.

We shall admit for the first system of the logic of acceptance, beside the classical logic infrastructure and the necessitation rule, a kind of K axiom drawn upon the *modus ponens* rule, for short, AK:

$$A(p \supset q) \supset (Ap \supset Aq) \qquad \text{(AK)}$$

We shall postulate the definitions:

D1. $Rp = A–p$
D2. $Tp = – Rp$
D3. $Cp = Ap \vee Rp$
D4. $Ip = – Cp$

The definitions D1–D4 introduce, one after the other, the rejection, the tolerability, the commitment and the irresolution or doubt.

To tolerate something means to not reject it. To be commited or resolute toward a question or a problem means to accept it or to reject it. To be irresolute or in doubt about a problem means to feel uncertain, to hesitate, not to be ready to take a decision. Irresolution is the opposite of commitment.

First we shall present a week variant of the axiomatic system of the acceptance with a single axiom, AK. We shall call this system AK. Syntactically, this system is analogous to the K system for alethic modalities. We have added some new definitions and we have proved some new theorems.

First we shall prove some derived rules, analogous to those proved by professors Hughes and Cresswell [17, …].

DR1. If $\alpha \supset \beta$ is a logical law, then $A\alpha \supset A\beta$ is a logical law too:

1. $\alpha \supset \beta$ hyp
2. $A(\alpha \supset \beta)$ (AN, 1)
3. $A(\alpha \supset \beta) \supset (A\alpha \supset A\beta)$, (RS, AK)
4. $A\alpha \supset A\beta$ (MP, 3, 2)

Another derived inference rule we shall prove, is :

DR2. If $\alpha \equiv \beta$ is a law of propositional logic, then $A\alpha \equiv A\beta$, will be a law of the logic of acceptance:

1. $\alpha \equiv \beta$ ip.
2. $\alpha \supset \beta$ (PL, 1)
3. $\beta \supset \alpha$ (PL, 1)
4. $A\alpha \supset A\beta$ (D R1, 2)

5. $A\beta \supset A\alpha$ (DR1, 3)
6. $A\alpha \equiv A\beta$ (PL, 4, 5)

Similarily , we shall prove the inference rule:

DR3. If $\alpha \equiv \beta$ is a law of the propositional logic, then $A(\gamma \vee \alpha) \equiv A(\gamma \vee \beta)$, will be a law of the logic of acceptance:

1.$\equiv \beta$ ip
2.$\gamma \vee \alpha \equiv \gamma \vee \beta$ (PL, 1)
3.$A(\gamma \vee \alpha) \equiv A(\gamma \vee \beta)$ (DR2,2)

In the system A of the logic of acceptance the following theorems will be proven:

A 1. $A(p \wedge q) \supset (Ap \wedge Aq)$

1. $(p \wedge q) \supset q$ (PL)
2. $A(p \wedge q) \supset Ap$ (DR1, 1)
3. $(p \wedge q) \supset q$ (P L)
4. $A(p \wedge q) \supset Aq$ (DR1, 3)
5. $A(p \wedge q) \supset (Ap \wedge Aq)$ (PL, 2, 4)

A 2. $(Ap \wedge Aq) \supset A(p \wedge q)$

1. $p \supset (q \supset (p \wedge q))$ (PL)
2. $Ap \supset A(q \supset (p \wedge q))$ (DR1,1)
3. $A(q \supset (p \wedge q)) \supset (Aq \supset A(p \wedge q))$ (SR, AK)
4. $Ap \supset (Aq \supset A(p \wedge q))$ (PLTranz, 2, 3)
5. $(Ap \wedge Aq) \supset A(p \wedge q)$ (PLImport, 4)

A 3. $A(p \wedge q) \equiv (Ap \wedge Aq)$

1. $A(p \wedge q) \equiv (Ap \wedge Aq)$ (PL, *A1, A2*)

A 4. $(Ap \vee Aq) \supset A(p \vee q)$

2. $p \supset (p \vee q)$ (PL)
3. $Ap \supset A(p \vee q)$ (DR1, 1)
4. $q \supset (p \vee q)$ (PL)
5. $Aq \supset A(p \vee q)$ (DR1, 3)
6. $(Ap \vee Aq) \supset A(p \vee q)$ (PL, 2, 4)

A 5. $A(p \supset q) \supset (Rq \supset Rp)$ (A MT)

1. $(p \supset q) \supset (-q \supset -p)$ (PL)
2. $A(p \supset q) \supset A(-q \supset -p)$ (DR1, 1)
3. $A(-q \supset -p) \supset (A-q \supset A-p)$ (SR, AK)
4. $A(p \supset q) \supset (A-q \supset A-p)$ (PLTranz, 2, 3)
5. $A(p \supset q) \supset (Rq \supset Rp)$ (ER, 4, D1)

A 6.	$(A(p \vee q) \wedge Rp) \supset Aq$	*(P Alt)*
1.	$((p \vee q) \wedge -p) \supset q$	(PL)
2.	$A((p \vee q) \wedge -p) \supset Aq$	(DR1, 1)
3.	$A((p \vee q) \wedge -p) \equiv A(p \vee q) \wedge A-p)$	(SR, A3)
4.	$(A(p \vee q) \wedge A-p) \supset Aq$	(ER, 2, 3)
5.	$(A(p \vee q) \wedge Rp) \supset Aq$	(ER, 4, D1)

A 7.	$(R(p \wedge q) \wedge Ap) \supset Rq$	(P Incop)
1.	$-(p \wedge q) \wedge p) \supset -q$	(PL)
2.	$A-(p \wedge q) \wedge p) \supset A-q$	(DR1, 1)
3.	$A(-(p \wedge q) \wedge p) \equiv (A- (p \wedge q) \wedge Ap)$	(SR, A3,)
4.	$(A-(p \wedge q) \wedge Ap) \supset A-q$	(ER, 2, 3)
5.	$(R(p \wedge q) \wedge Ap) \supset Rq$	(ER, 4, D1)

A 8.	$(A(p \supset q) \wedge A(q \supset r)) \supset A(p \supset r)$	(P Tranzit.)
1.	$((p \supset q) \wedge (q \supset r)) \supset (p \supset r)$	(PL)
2.	$A ((p \supset q) \wedge (q \supset r)) \supset A(p \supset r)$	(DR1, 1)
3.	$A ((p \supset q) \wedge (q \supset r)) \equiv A ((p \supset q) \wedge A(q \supset r))$	(SR, A3)
4.	$(A (p \supset q) \wedge A(q \supset r)) \supset A(p \supset r)$	(ER, 2, 3)

A 9.	$(A(\lambda \vee C) \wedge A(-\lambda \vee B)) \supset A(C \vee B)$	(Resol P)
1.	$((\lambda \vee C) \wedge (-\lambda \vee B)) \supset (C \vee B)$	(PL)
2.	$A((\lambda \vee C) \wedge (-\lambda \vee B)) \supset A(C \vee B)$	(DR1, 1)
3.	$A((\lambda \vee C) \wedge (-\lambda \vee B)) \equiv (A(\lambda \vee C) \wedge A-\lambda \vee B))$	(SR,A3)
4.	$(A(\lambda \vee C) \wedge A(-\lambda \vee B)) \supset A(C\vee B)$	(ER,2, 3)

A 10.	$Tp \equiv -A - p$	
1.	$p \equiv - -p$	(PL)
2.	$Ap \equiv - -Ap$	(SR, 1)
3.	$Ap \equiv - -A - -p$	(ER, 2, 1)
4.	$Rp \equiv A-p$	(D1)
5.	$-Rp \equiv -A-p$	(PL, 4)
6.	$Tp \equiv -A-p$	(ER, 5, D2)

A 11.	$Ap \equiv -T-p$	
1.	$-Tp \equiv - -A- p$	(PL, A10)
2.	$-T-p \equiv - -A- -p$	(SR, 1)
3.	$-T-p \equiv Ap$	(ER, 2, LP, duble neg.)
4.	$Ap \equiv -T-p$	(LP, 3)

A 12. $T(p \vee q) \equiv (T p \vee Tq)$

1. $A(-p \wedge -q) \equiv (A-p \wedge A-q)$	(SR, A3,)
2. $A-(p \vee q) \equiv (A-p \wedge A-q)$	(PL, De Morgan, 1)
3. $-A-(p \vee q) \equiv -(A-p \wedge A-q)$	(PL Neg ≡ , 2)
4. $-A-(p \vee q) \equiv -A-p \vee -A-q$	(PL, De Morgan, 3)
5. $T(p \vee q) \equiv (Tp \vee Tq)$	(E R, 4, A10)

We shall introduce a new derived rule :
RD4. If $\alpha \supset \beta$ is a logical law in propositional logic, then $M\alpha \supset M\beta$ is a logical law in the system AMP of the logic of acceptance.
Schematically, we shall have this by formula:

$\models \alpha \supset \beta \Rightarrow \models T\alpha \supset T\beta$

1. $\alpha \supset \beta$	ip.
2. $- \beta \supset -\alpha$	PL, Contrapos
3. $A-\beta \supset A-\alpha$	DR1, 2
4. $-A-\alpha \supset - A-\beta$	Contrapos, 3
5. $T\alpha \supset T\beta$	ER,4, A10

A 13. $T(p \supset q) \equiv (Ap \supset Tq)$

1. $T(-p \vee q) \equiv (T-p \vee Tq)$	(SR, A12)
2. $–Ap \equiv T-p$	(PL, A11)
3. $T(-p \vee q) \equiv (-Ap \vee Tq)$	(ER, 1,2)
4. $T(p \supset q) \equiv (Ap \supset Tq)$	(ER, 3)

A 14. $T(p \wedge q) \supset (Tp \wedge Tq)$

1. $(p \wedge q) \supset p$	(P L)
2. $T(p \wedge q) \supset Tp$	(DR4, 1)
3. $(p \wedge q) \supset q$	(PL)
4. $T(p \wedge q) \supset Tq$	(DR4, 3)
5. $T(p \wedge q) \supset (Tp \wedge Tq)$	(PL, 2, 4)

A 15. $A(p \vee q) \supset (Ap \vee Tq)$

1. $A(-q \supset p) \supset (A-q \supset Ap)$	(SR, AMP)
2. $A(q \vee p) \supset (-A-q \vee Ap)$	(PL, 1)
3. $A(p \vee q) \supset (Ap \vee -A-q)$	(PL, 2)
4. $A(p \vee q) \supset (Ap \vee Tq)$	(ER, 3, D1, D2)

6. THE D SYSTEM AND THE LOGIC OF ACCEPTANCE. THE AD SYSTEM

If we add to the AK system presented above a new axiom (see AD below) which requires for the agent to be consistent in his options or decisions, then we shall obtain a new normal system for the logic of acceptance, analogous to the standard deontic system D. I shall call it the AD system.

$A(p \supset q) \supset (Ap \supset Aq)$ (AK)
$-(Ap \wedge Rp)$ (AD)

The ANC axiom states that no opinion, offer, claim, pray, excuse, a.s.o. can be at the same time accepted and rejected. This axiom is analogous to von Wright's well known axiom $-(Op\wedge O\text{-}p)$ which prevents contradictory obligations. The ANC axiom describes an axiological principle which claims mutual consistency for our decisions or value judgements. On the basis of ANC axiom, definition D2 and extensionality rule ER, we can prove the theorem:

A 16. $Ap \supset Tp$

1. $–Ap \vee -Rp$ (PL, AK)
2. $Ap \supset -Rp$ (PL, 1)
3. $Ap \supset Tp$ (ER, 2, D2)

A 17. $T(p \vee -p)$

1. $A(p \vee -p) \supset T(p \vee -p)$ (SR, A16)
2. $p \vee -p$ (PL)
3. $A(p \vee -p)$ (Accept Rul., 2)
4. $T(p \vee -p)$ (MP, 1, 3)

A 18. $Tp \vee T\text{-}p$

1. $T(p \vee -p) \supset (Tp \vee T\text{-}p)$ (SR, A10)
2. $Tp \vee T\text{-}p)$ (MP, 1, A17)

A 19. $Ap \vee Rp \vee Ip$

1. $p \vee -p$ (PL)
2. $Dp \vee -Dp$ (SR, 1)
3. $Dp \vee Ip$ (ER,2, D4)
4. $Ap \vee Rp \vee Ip$ (ER, 3, D3)

A 20. $Ip \supset -Ap$

1. $p \vee -p \vee q$ (PL)
2. $Ap \vee -Ap \vee Rp$ (SR, 1)
3. $Ap \vee Rp \vee -Ap$ (PL, 2)
4. $Dp \vee -Ap$ (ER, 3, D3)
5. $–Dp \supset -Ap$ (PL, 4)
6. $Ip \supset -Ap$ (ER, 5, D4)

A 21. Ip ⊃ -Rp

1.	q ∨ p ∨ -p	(PL)
2.	Ap ∨ Rp ∨ -Rp	(SR, 1)
3.	Dp ∨ Tp	(ER, 2, D2,D3)
4.	–Ip ∨ Tp	(P L, 3, D3, D4)
5.	Ip ⊃ Tp	(PL, 4)
6.	Ip ⊃ -Rp	(ER, 5)

A 22. Tp ≡ Ap ∨ Ip

1.	Ip ⊃ Tp	(ER, T21, D3)
2.	Ap ⊃ Tp	(A16)
3.	(Ap ∨ Ip) ⊃ Tp	(PL, 1, 2)
4.	Rp ∨ Ap ∨ Ip	(PL, T19)
5.	–Rp ⊃ Ap ∨ Ip	(PL, 4)
6.	Tp ⊃ (Ap ∨ Ip)	(ER,5, D2)
7.	Tp ≡ Ap ∨ Ip	(PL, 3, 6)

A 23. Cp ≡ Ap ∨ Rp

1.	p ≡ p	(PL)
2.	Cp ≡ Cp	(SR, 1)
3.	Cp ≡ Ap ∨ Rp	(ER, 2, D3 9

The D system describes a normal theory of accepting or rejecting opinions, claims, offers, arguments, solutions a.s.o. In addition to the AK system, in D, the request of mutual consistency of the agent's commitments is explicily stated. In accordance with this system the accepted opinions, claims, offers, a.s.o. must be always tolerable. The logical laws must always be accepted or at least tolerated (see the inference AN rule). In relation to a given opinion, offer or argument, one might always adopt only one of the three possible attitudes: to accept it, to reject it or to remain irresolute or in doubt about it (see theorem A19). The commitment or the resolution means to accept or to reject an opinion or an offer (see theorem A23). Irresolution or doubt means not to accept and not to reject. Irresolution is a state of uncertainity or hesitation, a state of noncommitment. Irresolution implies tolerability (see theorem A22, step 1). Tolerability has two forms: acception or irresolution (see A22). Tolerability is the opposite of rejection (see definition D2).

In accordance with a given opinion, question or problem an agent may be in a state of commitment or resolution or in a state of doubt or irresolution.

The logic of acceptance describes an agent's possible attitudinal states. This theory does not describe the changes of his mental states, the transition from an initial state of irresolution or doubt to a resolution state when he will accept or reject an examined opinion, argument, claim or offer. Of course, it is possible to investigate also the opposite paths from accepting a considered opinion and after a lapse of time to be in doubt about it or just to reject it.

The above sketched theory is a monadic one which does not specify the agents, the conditions or the time. This theory does not specify the reason or the support of the acception or rejection of an opinion or argument.

The axiomatic theories presented above are formal and abstract theories. They are unpragmatized and unoperational theories. These theories are little relevant for the study of the causes or reasons of human beings' changes in their options, opinion or attitudes about states of affairs, goals or programs.

But the above theories and axiomatic systems describe un abstract frame and an initial logical basis for more precise and applied systems.

We can develop further the above presented theories by making use of other more complex normal modal systems, such as S4, B or S5 or by adopting a semantic approach and by defining adequate methods for decision of the well constructed formulas.

It is also necessary to enrich the initial alphabet and language by adding new symbols for agents, actional situations (initial, terminal, intermediary), by taking acount of the acts or operations (elementary and complex) performed by agents, by defining their abilities and operational aperture, their goals and obiligations. Further, we can study collective agents or organizations making use of indeterministic automata, their competitive and cooperative acts, feasible and unfeasible plans and capable or fitted agents to fulfil different plans [35–37].

The logical theory of acceptance becomes more stimulative from a theoretical point of view and for applications when we can take account the reason for which an agent, placed in a determined actional situation, chooses a given behaviour to reach an assumed goal or he or she becomes capable to choose only legal modes of conduct.

We are in need for a dynamic logic of acceptances. We are interested to explore the changes which interfere in the receiver agent's states of knowledge when he takes account of the argument emitted by his interlocutor, to see why he accepts a given argument and to see why he rejects another argument.

The logic of acceptance must be connected to the argumentation theory and to the logic of human action. The logic of acceptance may be interpreted as a moment and as a component part of the theory of argumentation. The argumentation *pro* and the argumentation *contra* are two major classes of arguments closely connected to the two main value jugements possible in a case.

But we can also see the logic of acceptance as part of the decision theory or as a formal axiology.

The logic of acceptance is in our view strictly related to the logic of human actions, especially with value assignment of the goals and programs and with the value judgement of the actual performed course of actions.

The logic of acceptance may be seen as a moment or part of the argumentation theory since each argument is accepted, rejected or left in doubt by its addressees. An argument is accepted if its conclusion can be derived from the commonly accepted knowledge basis, rejected if the opposite proposition is derived, or left in doubt if neither one case happened.

7. FINAL REMARKS

1. The present inquiry on the logic of acceptance proposes a modal approach in the logical theory of value judgement of opinions, proposals, offers, claims, prayers, arguments, plans, programs, a.s.o.
2. The logic of acceptance is an abstract, modal theory of value judgement. Our attempt may be interpreted as a first step towards a formal axiology or logical theory of value assignment. As we see in chapter 4, our essay can be regarded as a generalization of the epistemic and doxastic logics created by Jakko Hintikka, of the deontic logic created by Georg von Wright and of teleological systems proposed by us twenty years ago [see 32–35].
3. The AK axiomatic systems is an initial simple system. Theorems A1–A4 and A10–A15 are analogous to some theorems of the normal system K presented in the classical handbook for modal logic written by G.E. Hughes şi M. J. Cresswell. The AK system is a normal, monadic system, without agents, without stipulating actional states or conditions, without taking account of selected criteria for value assignment and acception, doubt or rejection. This is a very weak system, unpragmatized. But this system can be connected to our logical system of actions modeled on graphs and indeterministic automata transposed in logical programs. Similarly, systems AK and AD can be related and applied in our dymamic logical theory of argumentation. During the different stages of the argumentative process, the accepted set of propositions of the disputants will be in flux. The theory of argumentation may be continued with another dynamic theory, with the negotiation theory.
4. Theorems A5–A9 transpose in the logic of acceptance Stoa and Megaric schemata of inference and resolution principle. In these theorems interfere the modal concepts of acception, rejection and irresolution or doubt.
5. We see in chapter 3 and in chapter 5, theorem A9, that the logic of acceptance can be treated in close connection to relational data knowledge and automated theorem proving.
6. The present inquiry may be completed by taking into consideration other more complex modal systems such as T, S4, B, S5 or other anormal modal systems.
7. Logical theory of acceptance can be further developed by constructing new systems with higher degree of pragmatic or actionalist commitment than just the presented ones. We have studied several logical systems of acceptance endowed with agents, actional situations and making use of iterated modal operators.

8. We have considered in another work [47] the distinction between acceptation and acceptability, between a casual acceptation and systematic and deeply founded acceptability. I have proposed for the logic of acceptance a method of decision by tree. I found it very stimulating to study the correlations between speech acts of assertion, truth value, truth and false, acception, rejection and irresolution as well as the notions of sincerity, lie. There is here a very a large area of very stimulating questions and problems rich in theoretical and practical implications

REFERENCES

1. Aristotel, *Organon IV*, *Topica, Respingerile sofistice*, Editura Ştiinţifică, Bucureşti, 1963.
2. Ashley, Kevin D., *Modeling Legal Argument*, The MIT Press, 1990.
3. Bieltz, Petre, Gheorghiu, Dumitru, *Logică Juridică*, Editura ProTransilvania, Bucureşti, 1998.
4. Bieltz, Petre, Gheorghiu, Dumitru, *Logică şi argumentare,* Teora Educaţional 1999.
5. Bratko, Ivan, *Programming in Prolog for Artificial Intelligence*, Addison-Weslwy, 1986.
6. Cicero, Marcus Tullius, *Opere alese*, vol. I–III, Editura Univers, Bucureşti, 1973.
7. *Eemeren, Frans H.van, *Handbook of Argumentation Theory*, Amsterdam, 1987.
8. Eemeren, Frans H. van Grootendorst, Rob, *Argumentation, Communication, and Fallacies,* Lawrence Erlbaum Associates Publishers, Hillsdale, New Jersey, London, 1992.
9. Cristian Coarfa, Negotiation-Based Multiagent System, *Agent Based Approaches in Intelligent Systems*, “Politehnica” University, Bucharest, 13–16 July 1998.
10. Ellis, B.*Rational Belief Systems*. Oxford: Blackwell, 1979.
11. Enescu Gh. *Tratat de logică*, Editura Lider, 1997.
12. Gavriliu, Leonard, *Mic tratat de sofistică*, Editura Iri, 1996.
13. Genesereth, M.R., Nilsson, Nils, *Logical Foundation of Artificial Intelligence*, Morgan Kaufmann Publ., 1987.
14. *Gärdenfors, Petre, *Knowledge in Flux, Modeling the Dynamics of Epistemic States,* The MIT Press, 1988.
15. Guler Gh. *Retorică şi adevăr,* Editura “JUS-RBA”, Bucureşti, 1995.
16. Hintikka, J.K.K., *Knowledge and Belief*, Ithaca, N.Y.: Cornell University Press, 1962.
17. Hughes, G.M. and M.J. Cresswell *A New Introduction to Modal Logic.*
18. Nienhuys-Cheng, Shan-Hwei şi Wolf, de Roland, *Foundations of Inductive Logic Programming*, Springer, 1997.
19. Ioan, Petru, *Logică şi metalogică*, Junimea 1983, pp. 93–155.
20. Kahane, Howard, *Logic and Contemporary Rhetoric*, Wadsworth Publishing Company, 1991.
21. Kalinowski, George, *Introduction à la logique juridique*, LCDY, Paris, 1965.
22. Kleeme, S. C. *Introduction to Metamathematics*, D. van Nostrand Company 1952, Russian version 1957, pp. 332–340.
23. Levi, Isaac, *For the Sake of the Argument*, Ramsey Test Conditionals, Introductive Inference, and Nonmanotonic Reasoning, Cambridge University Press, 1996.
24. Livius, Titus, *Ab urbe condita*, vol. 1–2, Biblioteca pentru toţi.
25. *Elemente constructive de argumentare juridică*, Minerva, Bucureşti, 1976.
26. Maritain, Jacques, *An Introduction to Logic*, Sheed and Ward, London, 1937.
27. Meyer, Michel, *Logique et argumentation*, Bruxelles, 1991.

28. Mihai, Gheorghe, *Elemente constructive de argumentare juridică*, Editura Academiei Bucureşti, 1982.
29. Mihai, Gheorghe, Papaghiuc, Gheorghe*, Încercări asupra argumentării,* Editura Junimea, Iaşi, 1985.
30. Năstăşel, Eugen, Ursu, Ioana, *Argumentul sau despre cuvântul bine gândit*, Editura Ştiinţifică şi Enciclopedică, 1980.
31. *Plantin, Cristiana, *Essais sur l'Argumentation, Introduction linguistique à l'étude de la parole argumentative,* Edition Kime, 1990.
32. Popa, Cornel, *Teoria definiţiei*, Editura Ştiinţifică, 1972.
33. Popa, Cornel, *Teleologica sau teoria logică a scopurilor*, Revista de Filozofie*,* **5,** 1978.
34. Popa, Cornel, Formal Structure of Some Teleological Systems, Revue Roumaine de Sciences Sociales, Série de Philosophie et Logique, **3–4**, pp. 419–435.
35. Popa, Cornel, Formalna structura pewnych systemow teleologicznych, Praxiologia, **3,** 1979.
36. Popa, Cornel, *Teleologika, czili logiczna celow*, Prakseologia, Warszawa, 1980, **1** (73), pp. 5–23.
37. Popa, Cornel, *Logic of Cognitive Acceptances and the Hypotheses of Epistemic Automata*, in *Progress in Cybernetics and Systems Research*, Robert Trappl, Luigi Ricciardi and Gordon Pask Eds, Hemisphere Publishing Corporation, 1982, pp. 447–451.
38. Popa, Cornel, *Conceptul de cunoaştere*, in *Teoria cunoaşterii ştiinţifice,* Editura Academiei, 1982, pp. 87–124.
39. Popa, Cornel, *Logica predicatelor*, Editura Hyperion, 1992.
40. Popa, Cornel, *Teoria acţiunii şi logica formală,* Editura Ştiinţifică şi Enciclopedică, 1984.
41. Popa, Cornel, Florea, Adina, Magda Human Action, Automata and Prolog, *Economic Computation and Economic Cibernetics Studies and Research*, 1–4, 1995, pp. 33–52.
42. Popa, Cornel, *Logic, Formal Languages and Theory of Argumentation,* in Revue Roumaine des Sciences Juridiques, **VI,** *2*, 1995, pp. 205–220.
43. Popa, Cornel, (in collab.), *Expert System and Argumentation Theory*, Revue Roumaine des Sciences Juridiques, **VII(XL),** *1*, Janvier – Juin 1996, pp. 75–87.
44. Popa, Cornel, (in collab.), Automated Problem Solving and Argumentation, *Revue Roumaine des Sciences Juridiques*, VIII(XLI),1, Janvier–Juin 1997, pp. 97–108.
45. Popa, Cornel, *Logica simbolică şi bazele de cunoştinţe*, I, II, "Politehnica" University Bucharest, 1998, part IV, *Teoria argumentării*.
46. Popa, Cornel, *Logica acţiunii, programele şi conflictele interumane*, Revista Română de Informatică, 1999.
47. Popa, Cornel, *Logică şi metalogică*, Fundaţia România de mâine, 2000.
48. Popa, Cornel, *Logica acceptării, opiniile şi argumentarea*, in Analele Universităţii "Spiru Haret", Seria de Filosofie, in print.
49. Quintilian*, Arta oratorică*, I–III, Biblioteca pentru toţi, Editura Minerva, Bucureşti, 1974.
50. Ricoeur, Paul, *Eseuri de hermeneutică*, Humanitas, 1995.
51. Salomaa, Arto*, Formal Languages*, Academic Press, 1973.
52. Stoianovici, Drégan, Dinu, Teodor, Marga, Andrei, *Logica generală*, Editura Didactică şi Pedagogică, Bucureşti, 1991.
53. Thucydides, *Războiul peloponesiac*, Editura Ştiinţifică, Bucureşti, 1966.
54. Toulmin, S. The Use of Argument, Cambridge University Press, 1958.
55. Turner, Raymond, *Logic for Artificial Intelligence*, 1984.
56. Von Wright, Georg, Henrik, *Explanation and Understanding*, Routledge and Kegan Paul, 1971, also *Explicaţie şi înţelegere*, Humanitas, 1995.
57. Zaharie, Dorin şi Năstase, Pavel, *Sisteme expert de gestiune*, Editura Romcart, 1993 .
58. Ziembinski, Z., *Practical Logic*, D.Reidel Publ. Comp. Dordrecht, Holland, 1976.